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Abstract

摘要

We provide a pedagogical introduction to $N = 1$ supergravity/supersymmetry in relation to particle physics. The various steps in the construction of a generic $N = 1$ supergravity model are briefly described, and we focus on its low-energy supersymmetric limit. The conditions for supersymmetry and supergravity breaking are investigated, and realistic mechanisms suitable for particle physics identified. We then study the model-building aspects of ”softly broken” supersymmetric extensions of the Standard Model and discuss several of their phenomenological features.

我们针对粒子物理领域的 $N = 1$ 超引力/超对称提供一篇教学入门介绍。本文简要概述构建一般 $N = 1$ 超引力模型的各个步骤，并且重点讨论其低能超对称极限。本文研究了超对称与超引力破缺的条件，识别出适用于粒子物理的现实机制。随后本文研究了标准模型“软破缺”超对称扩展的模型构建方面，并讨论了它们的若干唯象特征。

Keywords

关键词

$N = 1$ supergravity/supersymmetry - Soft supersymmetry breaking - Gravity mediation · Goldstino/gravitino · MSSM ·R -parity · Electroweak symmetry - Flavor problem · Dark matter - Collider phenomenology - Grand unification

$N = 1$ 超引力/超对称 - 软超对称破缺 - 引力介导 · 戈德斯蒂诺/引力微子 · 最小超对称标准模型 (MSSM) ·R 宇称 · 电弱对称 - 味问题 · 暗物质 - 对撞机唯象学 - 大统一

Introduction: Symmetries in Particle Physics

引言: 粒子物理学中的对称性

Symmetry principles emerge as a fundamental tool in the description of the laws of physics. Since particle physics is usually understood in terms of a quantum field theory (QFT), these principles enable to classify elementary particles and constrain the form of possible interactions. For instance, elementary particles organize according to irreducible representations of the Poincaré group and can be labelled by the eigenvalues of its two Casimir operators, namely, their mass and their spin. Gauge interactions are associated with a compact Lie group. Spacetime and internal symmetries allow the construction of multiple particle physics models, in particular, the so-called Standard Model of particle physics, which seems in capacity to describe almost all the currently available high-energy physics data collected in collider experiments.

对称性原理是描述物理规律的基本工具。由于粒子物理学通常以量子场论 (QFT) 框架描述，这些原理可用于对基本粒子分类，并约束可能相互作用的形式。例如，基本粒子按庞加莱群的不可约表示分类，可由其两个卡西米尔算子的本征值 (即质量和自旋) 标记。规范相互作用对应一个紧致李群。时空对称性与内对称性可用于构建多种粒子物理模型，其中最典型的就是粒子物理标准模型，该模型似乎能够描述对撞机实验中目前收集到的几乎所有高能物理数据。

Since the properties of elementary particles are associated with the underlying symmetry group, one may wonder what type of symmetries are compatible with the basic underlying principles. In this respect, QFT places severe constraints on the possible structures. Indeed, if we assume that acceptable symmetries are encoded by Lie algebras, Coleman and Mandula [23] established that the most general form that the corresponding Lie algebras can take is (If we suppose only massless particles, a larger symmetry group can be obtained, namely, the conformal group.):

由于基本粒子的性质与底层对称群相关，人们自然会探究哪类对称性与基础底层原理相容。对此，量子场论对可能的结构给出了严格限制: 如果我们认可可接受的对称性由李代数描述，Coleman 和 Mandula[23] 证明了对应李代数的最一般形式为 (若仅考虑无质量粒子，可得到更大的对称群，即共形群。):

$$\mathfrak{g} = \mathrm{Iso}\,(1,3) \times \mathfrak{g}_c$$

where $I\mathfrak{so}\,(1,3)$ is the Poincaré algebra and $\mathfrak{g}_c$ a compact Lie algebra. All the generators of $\mathfrak{g}_c$ commute with those of $I\mathfrak{so}\,(1,3)$. In other words, all the associated symmetries, in particular those encoding the gauge interactions, are ”neutral” with respect to spacetime. A fundamental assumption behind the construction of $\mathfrak{g}$ is that it contains only generators of a bosonic nature. However, due to the spin-statistics and Nœther theorems, it is in fact possible to obtain conserved charges of fermionic nature, closing their own algebra through anticommutating relations and thus leading to new algebraic structures called Lie superalgebras. Within this latter framework, it is possible to extend the family of Poincaré-compatible symmetries in a non-trivial way [81, 89] , and only supersymmetric extensions (If we suppose only massless particles, a larger symmetry structure can be obtained, namely, the superconformal algebra.) of the Poincaré algebra are then admissible in QFT. The simplest (non-trivial) $N = 1$ supersymmetric extension of spacetime symmetries is generated by the generators of the Poincaré algebra $\left(L_{\mu\upsilon} = -L_{\upsilon\mu}, P_\mu, \mu, \upsilon = 0, \ldots, 3\right)$ and one Majorana spinor $\left(Q_\alpha, \bar{Q}_{\dot{\alpha}}, \alpha, \dot{\alpha} = 1, 2\right)$ (Following the standard conventions, indices for left-handed spinors are denoted by $\alpha, \beta, \ldots$; indices for right-handed spinors are taken to be $\dot{\alpha}, \dot{\beta}, \ldots$; and vector indices are given by $\mu, \upsilon, \ldots$). The non-trivial part of the algebra involving the fermionic charges is

其中 $I\mathfrak{so}(1,3)$ 是庞加莱代数，$\mathfrak{g}_c$ 是紧致李代数。$\mathfrak{g}_c$ 的所有生成元都与 $I\mathfrak{so}(1,3)$ 的生成元对易。换言之，所有相关对称性 (尤其是描述规范相互作用的对称性) 相对于时空都是“中性”的。构建 $\mathfrak{g}$ 的一个基本假设是，它仅包含玻色性的生成元。但根据自旋统计定理和诺特定理，实际上可以存在费米性的守恒荷，它们通过反对易关系闭合自身的代数，由此得到名为李超代数的新代数结构。在该框架下，可以非平凡地扩展与庞加莱相容的对称性族 [81, 89]，因此在量子场论中，只有庞加莱代数的超对称扩展 (若仅考虑无质量粒子，可得到更大的对称结构，即超共形代数。) 是可容许的。最简单的 (非平凡) $N=1$ 时空对称性超对称扩展由庞加莱代数的生成元 $\left(L_{\mu\upsilon}=-L_{\upsilon\mu}, P_\mu, \mu, \upsilon= 0,\ldots,3\right)$ 和一个马约拉纳旋量 $\left(Q_\alpha, \bar{Q}_{\dot{\alpha}}, \alpha, \dot{\alpha}=1,2\right)$ 生成 (遵循标准约定，左手旋量的指标记为 $\alpha, \beta, \ldots$；右手旋量的指标记为 $\dot{\alpha}, \dot{\beta}, \ldots$；矢量指标记为 $\mu, \upsilon, \ldots$)。该代数涉及费米荷的非平凡部分为

$$\left\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\right\} = -2\sigma^\mu_{\alpha\dot{\alpha}} P_\mu, \tag{1}$$

where $\sigma^\mu_{\alpha\dot{\alpha}}$ is defined below - see after Eq. (10). Thus, supersymmetry is a non-trivial extension of the Poincaré algebra.

其中 $\sigma^\mu_{\alpha\dot{\alpha}}$ 的定义见下文——参见式 (10) 之后。因此，超对称是庞加莱代数的非平凡扩展。

This discovery subsequently led to the introduction of supersymmetry in particle physics. Since Q_α is of fermionic nature, supersymmetry is a symmetry that maps a boson into a fermion and vice versa, and, as can be proved, supersymmetric multi-plets contain an equal number of bosonic and fermionic degrees of freedom.One may also promote the invariance under supersymmetry to a local version: since the anticommutator of two local supersymmetric transformations amounts to a spacetime translation (see Eq. 1), local supersymmetry in particular contains gravity and is thus called supergravity.

这一发现随后推动了粒子物理学中超对称的引入。由于 Q_α 具有费米性，超对称是一种将玻色子映射为费米子、反之亦然的对称性，并且可证，超对称多重态包含相等数目的玻色子和费米子自由度。还可以将超对称不变性推广为定域形式: 由于两个定域超对称变换的对易子等于一个时空平移 (参见式 1)，因此定域超对称本身包含引力，被称为超引力。

The purpose of this chapter is to describe applications of supergravity in particle physics. To this end, we shall first design a generic theory of fields invariant under $N=1$ supergravity. Then, given that exact supergravity is incompatible with the known particle spectrum, we will study the conditions for supersymmetry and supergravity breaking and present realistic mechanisms suitable for particle physics: we will more specifically highlight the case of ”gravity mediation,” where gravitational effects convey supersymmetry breaking from a ”hidden” sector to the observable one. Finally, we will discuss several phenomenological features of softly broken supersymmetric extensions of the Standard Model.

本章的目的是描述超引力在粒子物理学中的应用。为此，我们首先将构建一个在 $N=1$ 超引力下不变的通用场论。随后，考虑到严格超引力与已知粒子谱不兼容，我们将研究超对称和超引力破缺的条件，并介绍适用于粒子物理的实际机制: 我们将特别强调“引力介导”的情况，即引力效应将超对称破缺从“隐”sector 传递到可观测 sector。最后，我们将讨论标准模型的软破缺超对称扩展的若干唯象特征。

How to Build an Action in Supergravity

如何构造超引力中的作用量

The first theory of pure supergravity was obtained by Freedman, van Nieuwen-huizen, and Ferrara [70] and Deser and Zumino [31] independently. It took a couple of years to understand how matter and Yang-Mills fields could couple to supergravity, and the supergravity coupling of arbitrary interacting matter fields was derived in [7, 8, 24-27]. The first attempts to build invariant Lagrangians [24-27] were performed in the component formalism or using appropriate tensor calculus techniques [60, 61, 141-143]. Then, it was realized that a superspace suited to supergravity model building could be introduced [1,6,16,85,152-154,161]. The key advantage of this superspace rests with the automatic invariance under supergravity underlying this formalism, calling all the necessary auxiliary fields and resulting in a suitable action There exist several books [13, 17, 69, 69, 71, 73, 75, 118, 121, 146 , 148, 150, 151, 155] and reviews [14, 42, 122, 123] on the topic.

纯超引力的第一个理论由弗里德曼、范·纽文惠曾和费拉拉 [70] 与德塞尔、祖米诺 [31] 独立得到。人们花了数年时间才弄清楚物质场和杨-米尔斯场如何与超引力耦合，任意相互作用物质场的超引力耦合在文献 [7, 8, 24-27] 中被推导出来。构造不变拉格朗日量的最初尝试 [24-27] 是在分量形式框架下或使用适当的张量演算技术完成的 [60, 61, 141-143]。随后人们意识到，可以引入适合超引力模型构建的超空间 [1,6,16,85,152-154,161]。这种超空间的核心优势在于，该形式体系本身就自带超引力下的自动不变性，涵盖了所有必要的辅助场并得到合适的作用量。关于该主题已有多部著作 [13, 17, 69, 69, 71, 73, 75, 118, 121, 146 、148, 150, 151, 155] 和综述 [14, 42, 122, 123] 。

The purpose of the current section is to provide the reader with the salient points which lead from general principles to the four-dimensional $N = 1$ supergravity Lagrangian. The construction closely follows the book of Wess and Bagger [151] and is based on the recent book and review by one of the authors [45, 148]. In particular, these two last references are complementary since [148] offers a pedagogical introduction to supergravity, providing many technical details, while technical details are omitted in [45], with a focus on the conceptual scheme that leads to the four-dimensional $N = 1$ supergravity Lagrangian.

本节的目的是向读者介绍从一般原理得到四维 $N = 1$ 超引力拉格朗日量的核心要点。本次构造紧密遵循韦斯和巴格的著作 [151]，并且基于本文其中一位作者近期的著作与综述 [45, 148]。特别地，这两份参考文献是互补的:[148] 提供了超引力的入门教学，给出了大量技术细节；而 [45] 省略了技术细节，聚焦于推导出四维 $N = 1$ 超引力拉格朗日量的概念框架。

Four steps can be identified in the derivation of the supergravity-invariant Lagrangian. The first one is purely geometrical and consists in extending the notion of superspace, already considered in supersymmetry model building [62, 134, 135], but now in the context of supergravity, i.e., constructing a curved superspace [17,85, 152-154]. This curved superspace is such that, at each superspace point, a tangent space exists, behaving like the traditional flat superspace of global supersymmetry. The corresponding structure group is thus the Lorentz group. In particular, in the Einstein frame, one has an invariance under the superdiffeomorphisms, whereas in the tangent or Lorentz frame, one has local Lorentz invariance.

推导超引力不变拉格朗日量可以分为四个关键步骤。第一步是纯几何层面的: 扩展已经用于超对称模型构建的超空间概念 [62, 134, 135]，将其应用到超引力场景中，也就是构造弯曲超空间 [17,85, 152-154]。这种弯曲超空间满足: 超空间的每一点都存在一个切空间，其性质和整体超对称性的传统平坦超空间一致，因此对应的结构群是洛伦兹群。具体来说，在爱因斯坦框架下，理论具有超微分同胚不变性；而在切空间/洛伦兹框架下，理论具有局域洛伦兹不变性。

Following the standard techniques of general relativity, two dynamical variables (which are indeed two superfields) are introduced. The first dynamical superfield is the supervierbein, which connects components in the Einstein frame to components in the Lorentz (or flat) frame. The second dynamical variable is the supercon-nection, which enables to define covariant derivatives. Then, one associates (after computing the (anti)commutators of covariant derivatives) two superfields with the two dynamical variables: the torsion and curvature tensors. Some constraints are imposed to the torsion tensor in order to dispose of overabundant degrees of freedom [85, 151, 152, 154]. These constraints are of physical importance since, in addition to drastically reducing the number of degrees of freedom, they allow to explicitly construct physical models in particle physics that are invariant under supergravity and supersymmetric transformations at low energy.

遵循广义相对论的标准方法，我们引入两个动力学变量 (它们实际上是两个超场)。第一个动力学超场是超 vierbein(超标架)，用来关联爱因斯坦框架和洛伦兹 (平坦) 框架下的分量。第二个动力学变量是超联络，它允许我们定义协变导数。在计算完协变导数的 (反) 对易子后，我们可以给这两个动力学变量对应两个超场: 挠率张量和曲率张量。为了消去多余的自由度，我们需要对挠率张量施加一些约束 [85, 151, 152, 154]。这些约束具有重要的物理意义: 除了能大幅减少自由度数量，它们还允许我们明确构造出在低能下满足超引力不变性和超对称变换的粒子物理物理模型。

Requiring torsion constraints, the Bianchi identities lead to 13 equations. Solving these equations allows to express all torsion and curvature tensors in terms of three basic superfields [41,86,136,151]: a chiral superfield $\mathcal{R}$, a chiral symmetric spinor superfield $W_{(\alpha\beta\gamma)}$, and a real vector superfield G_μ . It was observed in [96] that the structure of constraints upon the torsion tensors is invariant under superconformal or Howe-Tucker transformations. Superconformal transformations are the supergravity analogue of Weyl transformations, or rescaling of the metric, in general relativity and play a central rôle in the construction of a correctly normalized action in particle physics.

施加挠率约束后，比安基恒等式会给出 13 个方程。求解这些方程就可以将所有挠率和曲率张量用三个基本超场表示出来 [41,86,136,151]: 手征超场 $\mathcal{R}$ 、手征对称旋量超场 $W_{(\alpha\beta\gamma)}$ 和实矢量超场 G_μ 。文献 [96] 指出，挠率张量的约束结构在超共形变换 (即豪-塔克变换) 下保持不变。超共形变换是超引力版本的广义相对论外尔变换 (也就是度规缩放变换)，它在构造粒子物理中归一化正确的作用量时发挥核心作用。

Having expressed all torsion and curvature tensors in terms of the three basic superfields, and using the large symmetry due to the supergravity algebra, many components can be set to zero by means of an appropriate choice of the parameters [143, 151, 152] . In particular, for the supervierbein and the superfields $\mathcal{R}$ and G_μ , we have

在将所有挠率和曲率张量用三个基本超场表示后，利用超引力代数带来的大对称性，通过恰当选择参数 [143, 151, 152] 可将众多分量置零。特别地，对于超四标架以及超场 $\mathcal{R}$ 和 G_μ ，我们有

$$E_{\widetilde{M}}{}^{M}(z)\Big| = \begin{pmatrix} e_{\widetilde{\mu}}{}^{\mu}(x) & \frac{1}{2}\psi_{\widetilde{\mu}}{}^{\alpha}(x) & \frac{1}{2}\overline{\psi}_{\widetilde{\mu}\dot{\alpha}}(x) \\ 0 & \delta_{\dot{\alpha}}{}^{\alpha} & 0 \\ 0 & 0 & \delta^{\dot{\alpha}}{}_{\dot{\alpha}} \end{pmatrix},\ G_{\mu}(z)\Big| = -\frac{1}{6}M(x), \tag{2}$$

where $z \equiv \left(x, \theta, \overline{\theta}\right)$ corresponds to a point in a superspace and $X\,|$ represents the lowest-order component of the superfield X in its expansion in terms of the Grassmann variables $\theta, \overline{\theta}$ (Lorentz indices in flat space are taken to be untilded $M = (\mu, \alpha, \dot{\alpha})$, while Einstein indices in curved space are taken to be tilded $\widetilde{M} = (\widetilde{\mu}, \widetilde{\alpha}, \widetilde{\dot{\alpha}})$. The three entries are related to the vector, the left-handed spinor, and the right-handed spinor counterpart of a point in the superspace, i.e., the fermionic counterpart of the spacetime position x in the superspace). The fields of the supergravity multiplet are then the helicity-2 graviton $e_{\widetilde{\mu}}{}^{\mu}$, the helicity-3/2 gravitino $\left(\psi_{\widetilde{\mu}}{}^{\alpha}(x), \overline{\psi}_{\widetilde{\mu}\dot{\alpha}}(x)\right)$ (which is a Majorana spinor-vector), and two auxiliary fields, M a complex scalar and b_{μ} a real vector. The last field of the gravity multiplet, the connection, is a composite field and can be expressed in terms of the graviton and the gravitino [31, 70] .

其中 $z \equiv \left(x, \theta, \overline{\theta}\right)$ 对应超空间中的一个点，$X\,|$ 表示超场 X 按格拉斯曼变量 $\theta, \overline{\theta}$ 展开时的最低阶分量 (平坦空间的洛伦兹指标不带波浪号 $M = (\mu, \alpha, \dot{\alpha})$ ，弯曲空间的爱因斯坦指标带波浪号 $\widetilde{M} = (\widetilde{\mu}, \widetilde{\alpha}, \widetilde{\dot{\alpha}})$ 。这三个分量分别对应超空间中点的矢量、左手旋量和右手旋量对应部分，即超空间中时空位置 x 的费米对应部分)。超引力多重态的场分别是螺旋度为 2 的引力子 $e_{\widetilde{\mu}}{}^{\mu}$ 、螺旋度为 3/2 的引力微子 $\left(\psi_{\widetilde{\mu}}{}^{\alpha}(x), \overline{\psi}_{\widetilde{\mu}\dot{\alpha}}(x)\right)$ (它是马约拉纳旋量矢量)，以及两个辅助场: M 一个复标量和 b_{μ} 一个实矢量。引力多重态的最后一个场即联络是复合场，可以用引力子和引力微子 [31, 70] 表示。

If we consider a general transformation, combining an arbitrary superdiffeo-morphism and a local Lorentz transformation, there is no reason why the gauge fixing conditions above should be preserved. This means that the set of all possible transformations must be restricted to the subset protecting the previous gauge condition [151-153]. This restricted set of transformations is called supergravity transformations. On the one hand, a supergravity transformation is parameterized by a general transformation in the fermionic direction of the superspace, corresponding to a local supersymmetric transformation. On the other hand, the transformation in the bosonic direction of the superspace and the local Lorentz transformation are related to the transformation in the fermionic direction (for the Lorentz transformation, it also involves the auxiliary fields M and b_{μ}).

如果我们考虑结合任意超微分同胚和局域洛伦兹变换的一般变换，上述规范固定条件没有理由会被保持。这意味着所有可能变换的集合必须限制在能保留已有规范条件的子集内 [151-153]。这个受限的变换集合就被称为超引力变换。一方面，超引力变换由超空间费米方向上的一般变换参数化，对应局域超对称变换。另一方面，超空间玻色方向的变换和局域洛伦兹变换都与费米方向的变换相关 (对于洛伦兹变换，它还涉及辅助场 M 和 b_{μ})。

Using firstly the constraints on the torsion tensors and secondly the explicit relationship between the torsion or curvature tensors with the supervierbein and the superconnection (expressed in the gauge (2)) allows to compute the lowest components of the basic superfields $\mathcal{R}$, $W_{(\alpha\beta\gamma)}$, and G_{μ} together with the lowest components of their derivatives with respect to the covariant derivatives [151]. It turns out that all these lowest components, or the lowest components of derivatives, are expressed in terms of the supergravity multiplet $e_{\widetilde{\mu}}{}^{\mu}, \psi_{\widetilde{\mu}}{}^{\alpha}, M, b_{\mu}$.

先利用挠率张量上的约束，再利用挠率/曲率张量与超四标架和超联络 (取规范 (2) 下的形式) 之间的显式关系，就可以计算基本超场 $\mathcal{R}, W_{(\alpha\beta\gamma)}$ 和 G_μ 的最低分量，以及它们对协变导数导数的最低分量 [151]。可以发现，所有这些最低分量，或是导数的最低分量，都可以用超引力多重态 $e_{\tilde{\mu}}{}^{\mu}, \psi_{\tilde{\mu}}{}^{\alpha}, M, b_\mu$ 表示。

Having fixed the gauge, defined what is called supergravity transformations, and obtained all physical quantities in terms of the supergravity multiplet, one should then compute the transformation of the supergravity multiplet under supergravity transformations [60, 65, 143, 151, 153] .

在完成规范固定、定义了所谓的超引力变换、并得到所有用超引力多重态表示的物理量之后，接下来就需要计算超引力变换下超引力多重态的变换 [60, 65, 143, 151, 153] 。

The second step in the construction consists in introducing superfields in the curved superspace. As in supersymmetry, two types of superfields will be considered in applications of supergravity in particle physics: chiral superfields associated with ordinary matter and vector superfields associated with Yang-Mills theory. It is remarkable that the supersymmetry concepts of chiral and vector superfields extend to the supergravity context. This is in fact a consequence of the torsion constraints, which amount to an integrability condition. Chiral and vector multiplets were independently introduced in supergravity within the tensor calculus approach [61, 141, 142] or the superspace approach [153] or when constructing the first invariant Lagrangians [24-27, 63-65].

构造的第二步是在弯曲超空间中引入超场。和超对称理论一样，在粒子物理的超引力应用中通常会考虑两类超场: 对应普通物质的手征超场，以及对应杨-米尔斯理论的矢量超场。值得注意的是，手征和矢量超场的超对称概念可以直接推广到超引力框架中。这实际上是挠率约束的结果，挠率约束本质上是一个可积性条件。手征和矢量多重态是人们在研究超引力时独立引入的，相关工作包括张量演算方法 [61, 141, 142]、超空间方法 [153]，以及构造第一批不变拉格朗日量的研究 [24-27, 63-65]。

Superfields live in the curved superspace, but we need to compute their covariant derivatives with respect to the variables with Lorentz indices. For instance, introducing $\mathcal{D}_\alpha, \overline{\mathcal{D}}_{\dot{\alpha}}$ (pay attention to the fact that these are indices in the Lorentz frame and not indices in the Einstein frame), the covariant derivatives with respect to left- and right-handed spinors, a chiral superfield is defined by the chirality condition (For the spinor scalar products, we take the usual conventions, namely, $\chi \cdot \chi = \chi^\alpha \chi_\alpha$ for left-handed spinors and $\overline{\chi} \cdot \overline{\chi} = \overline{\chi}_{\dot{\alpha}} \overline{\chi}^{\dot{\alpha}}$ for right-handed spinors.)

超场定义在弯曲超空间上，但我们需要计算它们对洛伦兹指标变量的协变导数。例如，引入 $\mathcal{D}_\alpha, \overline{\mathcal{D}}_{\dot{\alpha}}$ (注意这些指标是洛伦兹系中的指标，而非爱因斯坦系中的指标)，对手征超场关于左手和右手旋量求协变导数后，可通过手征性条件定义手征超场 (对于旋量标积，我们采用常规约定，即左手旋量为 $\chi \cdot \chi = \chi^\alpha \chi_\alpha$ ，右手旋量为 $\overline{\chi} \cdot \overline{\chi} = \overline{\chi}_{\dot{\alpha}} \overline{\chi}^{\dot{\alpha}}$ 。)

$$\overline{\mathcal{D}}_{\dot{\alpha}} \Phi = 0 \tag{3}$$

$$\varphi = \Phi \Big| , \ \chi_\alpha = \frac{1}{\sqrt{2}} \mathcal{D}_\alpha \Phi \Big| , \ F = \frac{1}{4} \mathcal{D} \cdot \mathcal{D} \Phi \, | \, . \tag{4}$$

where the second line explicitly provides the components in this gauge. They correspond to a scalar field

φ , a left-handed spinor χ , and an auxiliary field F . A vector superfield is defined by the reality condition $V^\dagger = V$. As in supersymmetry, it is possible to select the Wess-Zumino gauge defined by

其中第二行明确给出了该规范下的各分量，对应一个标量场 φ 、一个左手旋量 χ 和一个辅助场 F 。矢量超场由实性条件 $V^\dagger = V$ 定义。和超对称理论一样，我们可以选取如下定义的韦斯-祖米诺规范：

$$V| = 0, \mathcal{D}_\alpha V| = 0, \overline{\mathcal{D}}_{\dot\alpha} V| = 0, \mathcal{D} \cdot \mathcal{D}\Phi| = 0, \overline{\mathcal{D}}_{\dot\alpha}\overline{\mathcal{D}}_{\dot\Phi}| = 0. \tag{5}$$

For both chiral and vector superfields, in order to obtain the full supergravity action, we have to compute several (lowest-order components of) derivatives. This computation is performed using the algebra of supergravity (the (anti)commutators of covariant derivatives expressed in terms of the superfields $\mathcal{R}, W_{(\alpha\beta\gamma)}$, and G_μ).

对于手征超场和矢量超场，为了得到完整的超引力作用量，我们需要计算多个导数的最低阶分量。这个计算可以利用超引力代数完成，也就是用超场 $\mathcal{R}, W_{(\alpha\beta\gamma)}$ 和 G_μ 表示的协变导数的对易子/反对易子。

At this point, we would like to stress on a fundamental property of chiral superfields. If $\Phi^\dagger$ is an anti-chiral superfield, then it can be shown that [74, 153] $\mathcal{D}_{\dot\alpha}\left(\overline{\mathcal{D}}\cdot\overline{\mathcal{D}} - 8\mathcal{R}\right)\Phi^\dagger = 0$ or equivalently the superfield

在此我们需要强调手征超场的一个基本性质：如果 $\Phi^\dagger$ 是反手征超场，那么可以证明 [74, 153] $\mathcal{D}_{\dot\alpha}\left(\overline{\mathcal{D}}\cdot\overline{\mathcal{D}} - 8\mathcal{R}\right)\Phi^\dagger = 0$ ，即等价的超场

$$\Xi = \left(\overline{\mathcal{D}}\cdot\overline{\mathcal{D}} - 8\mathcal{R}\right)\Phi^\dagger \tag{6}$$

is chiral. This property is central in obtaining a compact chiral expression of the supergravity action.

是手征的。这个性质对得到超引力作用量的紧致手征表达式至关重要。

Vector superfields can be introduced to describe any Yang-Mills interactions. Consider a compact Lie algebra $\mathfrak{g}_c$ of dimension n and a unitary representation $\mathfrak{R}$, and denote the generators of $\mathfrak{g}_c$ in the representation $\mathfrak{R}$ by $T_a = T_a^\dagger, a = 1, \ldots, n$. If $f_{ab}{}^c \in \mathbb{R}$ are the structure constants of $\mathfrak{g}_c$, the Lie brackets take the form

我们可以引入矢量超场来描述任意杨-米尔斯相互作用。考虑维数为 n 的紧致李代数 $\mathfrak{g}_c$ 以及幺正表示 $\mathfrak{R}$ ，将表示 $\mathfrak{R}$ 下 $\mathfrak{g}_c$ 的生成元记为 $T_a = T_a^\dagger, a = 1, \ldots, n$ 。如果 $f_{ab}{}^c \in \mathbb{R}$ 是 $\mathfrak{g}_c$ 的结构常数，则李括号可以写为

$$[T_a, T_b] = i f_{ab}{}^c T_c.$$

Associated with the vector superfield $V = V^a T_a$, we define the spinor superfield strength tensor

我们关联矢量超场 $V = V^a T_a$ ，定义旋量超场强度张量为

$$\mathcal{W}_\alpha = -\frac{1}{4}\left(\overline{\mathcal{D}}\cdot\overline{\mathcal{D}} - 8\mathcal{R}\right)e^{2gV}\mathcal{D}_\alpha e^{-2gV},$$

where g is the coupling constant. Observe that $\mathcal{W}_\alpha$ is a chiral spinor superfield because of the "projection" operator $\overline{\mathcal{D}} \cdot \overline{\mathcal{D}} - 8\mathcal{R}$. Again, the algebra of supergravity enables to compute (using (5)) $\mathcal{W}_\alpha \left|, \mathcal{D}_\beta \mathcal{W}_\alpha\right|$, and $\overline{\mathcal{D}} \cdot \overline{\mathcal{D}} \mathcal{W}_\alpha \mid$. It turns out that the degrees of freedom of a vector superfield are $v_\mu, \left(\lambda_\alpha, \bar{\lambda}_{\dot{\alpha}}\right)$, and D , i.e., a vector, a Majorana spinor, and a real scalar.

其中 g 为耦合常数。可见 $\mathcal{W}_\alpha$ 是手征旋量超场，这由投影算符 $\overline{\mathcal{D}} \cdot \overline{\mathcal{D}} - 8\mathcal{R}$ 保证。超引力代数同样可通过 (5) 计算得到 $\mathcal{W}_\alpha \left|, \mathcal{D}_\beta \mathcal{W}_\alpha\right|$ 和 $\overline{\mathcal{D}} \cdot \overline{\mathcal{D}} \mathcal{W}_\alpha \mid$ 。结果表明，向量超场的自由度为 $v_\mu, \left(\lambda_\alpha, \bar{\lambda}_{\dot{\alpha}}\right)$ 和 D ，即一个向量、一个马约拉纳旋量和一个实标量。

We can now assume that the chiral superfield Φ is in the representation $\mathfrak{R}$ of $\mathfrak{g}_c$, i.e., transforms like $\Phi \to e^{-2gi\Lambda}\Phi$, where $\Lambda = \Lambda^a T_a$ and Λ^a are chiral superfields. Of course, the conjugate of a chiral superfield, which is an anti-chiral superfield and whose fermionic component contains a right-handed fermion, is in the representation $\overline{\mathfrak{R}}$, the complex conjugate representation of $\mathfrak{R}$, with generators $-T_a^\star = -T_a^t$. It is possible to couple the chiral superfield Φ to the vector superfield [148, 151] in an invariant way by considering $\mathcal{X} = \left(\overline{\mathcal{D}} \cdot \overline{\mathcal{D}} - 8\mathcal{R}\right)\Phi^\dagger e^{-2gV}$ (the analogue of (6)) which is obviously chiral and is again obtained by computing the lowest-order components of its derivatives up to the order 2. This computation is performed using the algebra of supergravity, but in the so-called Wess-Zumino gauge for which $V^3 = 0$.

现在我们可以假设手征超场 Φ 属于 $\mathfrak{g}_c$ 的表示 $\mathfrak{R}$ ，即变换性质与 $\Phi \to e^{-2gi\Lambda}\Phi$ 一致，其中 $\Lambda = \Lambda^a T_a$ 和 Λ^a 均为手征超场。显然，手征超场的共轭是反手征超场，其费米子分量包含右旋费米子，属于表示 $\overline{\mathfrak{R}}$, 即 $\mathfrak{R}$ 的复共轭表示，生成元为 $-T_a^\star = -T_a^t$ 。我们可以通过构造 $\mathcal{X} = \left(\overline{\mathcal{D}} \cdot \overline{\mathcal{D}} - 8\mathcal{R}\right)\Phi^\dagger e^{-2gV}$ 将手征超场 Φ 以不变方式耦合到向量超场 [148, 151] ，这是式 (6) 的类比，$\mathcal{X} = \left(\overline{\mathcal{D}} \cdot \overline{\mathcal{D}} - 8\mathcal{R}\right)\Phi^\dagger e^{-2gV}$ 显然是手征的，同样是通过计算导数直到二阶的最低阶分量得到的。该计算使用超引力代数完成，但采用所谓的韦斯-祖米诺规范，该规范下满足 $V^3 = 0$ 。

Up to now, we have defined the gravity multiplet $\left(e_{\tilde{\mu}}{}^{\mu}, \psi_{\tilde{\mu}}{}^{\alpha}, \overline{\psi}_{\tilde{\mu}\dot{\alpha}}, M, b_\mu\right)$, the matter multiplets $\left(\varphi^i, \chi^i, F^i\right)$, $i = 1, \ldots, \dim \mathcal{R}$, and the Yang-Mills multiplets $\left(v_\mu^a, \lambda_\alpha^a, \overline{\lambda}_{\dot{\alpha}}^a, D^a\right)$, $a = 1, \ldots, \dim \mathfrak{g}_c$: these can finally be combined to construct a supergravity action. As seen in (4), a chiral superfield is defined in the curved superspace, but the constraint (3) and its components (4) involve covariant derivatives with respect to Lorentz indices. This, of course, will make the computation cumbersome. The idea of Julius Wess, which can be seen as a tour de force [84], consists in introducing new Θ -variables carrying a Lorentz index instead of an Einstein index, in such a way that the expansion of the chiral superfield Φ reduces to [151, 154]

到目前为止，我们已经定义了引力多重态 $\left(e_{\tilde{\mu}}{}^{\mu}, \psi_{\tilde{\mu}}{}^{\alpha}, \overline{\psi}_{\tilde{\mu}\dot{\alpha}}, M, b_\mu\right)$ 、物质多重态 $\left(\varphi^i, \chi^i, F^i\right)$, $i = 1, \ldots$ 、$\dim \mathcal{R}$ 以及杨-米尔斯多重态 $\left(v_\mu^a, \lambda_\alpha^a, \overline{\lambda}_{\dot{\alpha}}^a, D^a\right)$, $a = 1, \ldots, \dim \mathfrak{g}_c$: 最终可以将它们组合起来构造超引力作用量。如式 (4) 所示，手征超场定义在弯曲超空间中，但约束条件 (3) 及其分量形式 (4) 包含洛伦兹指标的协变导数。这自然会让计算变得十分繁琐。朱利叶斯·韦斯的思路堪称杰作 [84]，他引入携带洛伦兹指标而非爱因斯坦指标的新 Θ 变量，使得手征超场 Φ 的展开可以简化为 [151, 154]

$$\Phi(x, \Theta) = \Phi \left| + \sqrt{2}\Theta \cdot (\mathcal{D}\Phi) \right| - \frac{1}{4}\Theta \cdot \Theta (\mathcal{D} \cdot \mathcal{D}\Phi) \Big| = \varphi + \sqrt{2}\Theta \cdot \chi - \Theta \cdot \Theta F.$$

Next, in general relativity, the invariant measure of integration involves the determinant of the vierbein. Similarly, in curved superspace, the invariant measure of integration involves the superdeterminant of the supervierbein. However, in the context of a chiral form of the Lagrangian (see below), an adapted invariant measure, involving the capacity $\mathcal{E}$, has to be considered [6, 119, 122] .

接下来，在广义相对论中，积分的不变测度涉及 vierbein(Vierbein，四标架) 的行列式。类似地，在弯曲超空间中，积分的不变测度涉及超四标架的超行列式。然而，在拉格朗日量的手征形式下 (见下文)，我们必须考虑包含容量 $\mathcal{E}$ 的适配不变测度 [6, 119, 122] 。

We now have all the necessary material to compute the supergravity action, exploiting the notations introduced above. In the fourth and final step, we set $\mathfrak{g}_c$ to the specific algebra corresponding to the desired compact Lie group which determines the Yang-Mills interactions. The Lie algebra $\mathfrak{g}_c$ can be semisimple (including the product of simple Lie algebras) or can contain some abelian factors or $\mathfrak{u}(1)$ -terms. One introduces $\dim \mathfrak{g}_c$ vector superfields associated with $\mathfrak{g}_c$. Next, selecting a matter content in a unitary representation $\mathfrak{R}$ of $\mathfrak{g}_c$ (not necessarily irreducible), we associate $\dim \mathfrak{R}$ chiral fields in the representation $\mathfrak{R}$ and $\dim \mathfrak{R}$ anti-chiral fields in the representation $\overline{\mathfrak{R}}$. We collectively denote Φ^i and $\Phi^\dagger_{i*}$ as chiral or anti-chiral superfields, respectively. Further, we introduce three gauge-invariant functions: (1) the superpotential $W(\Phi)$, a holomorphic function depending on chiral superfields; (2) the gauge kinetic function $h_{ab}(\Phi)$ (where a, b are gauge indices in the adjoint representation of $\mathfrak{g}_c$), a holomorphic function depending on chiral superfields; and (3) the Kähler potential $K\left(\Phi, \Phi^\dagger\right)$, a real function depending on Φ and $\Phi^\dagger$. The Lagrangian (in chiral form) takes the form [77,151]

利用上文引入的记号，我们现在已经掌握了计算超引力作用量所需的全部基础内容。在第四步也是最后一步，我们将 $\mathfrak{g}_c$ 设定为对应目标紧致李群的特定代数，该李群决定了杨-米尔斯相互作用。李代数 $\mathfrak{g}_c$ 可以是半单的 (包括单李代数的直积)，也可以包含若干阿贝尔因子或 $\mathfrak{u}(1)$ 项。我们引入 $\dim \mathfrak{g}_c$ 个与 $\mathfrak{g}_c$ 关联的矢量超场。随后，选取属于 $\mathfrak{g}_c$ 的幺正表示 $\mathfrak{R}$ (不必是不可约表示) 的物质内容，我们将 $\dim \mathfrak{R}$ 个手征场对应到表示 $\mathfrak{R}$ ， 将 $\dim \mathfrak{R}$ 个反手征场对应到表示 $\overline{\mathfrak{R}}$ 。我们将手征超场和反手征场分别统一记为 Φ^i 和 $\Phi^\dagger_{i*}$ 。此外，我们引入三个规范不变函数:(1) 超势 $W(\Phi)$ ，它是依赖于手征超场的全纯函数；(2) 规范动力学函数 $h_{ab}(\Phi)$ (其中 a, b 是 $\mathfrak{g}_c$ 伴随表示下的规范指标)，它同样是依赖于手征超场的全纯函数；(3) 凯勒势 $K\left(\Phi, \Phi^\dagger\right)$ ，它是依赖于 Φ 和 $\Phi^\dagger$ 的实函数。手征形式的拉格朗日量可写为 [77,151]

$$\mathcal{L}_{\text{SUGRA}} = \int \mathrm{d}^2\Theta\mathcal{E}\left\{\frac{3m_p^2}{8}\left(\overline{\mathcal{D}}\cdot\overline{\mathcal{D}} - 8\mathcal{R}\right)e^{-\frac{1}{3m_p^2}K\left(\Phi,\Phi^\dagger e^{-2gV}\right)} \right.$$
$$\left. + W(\Phi) + \frac{1}{16g^2}h_{ab}(\Phi)\,\mathcal{W}^{a\alpha}\mathcal{W}^b_\alpha\right\} + \text{ h.c.,} \tag{7}$$

where m_p is the Planck mass. To make it more explicit, we have to expand all fields in the Θ -variables. Since all superfields have a large number of components, this computation is lengthy, but not fundamentally complicated. Once the expansion has been performed, we have to eliminate the auxiliary fields F^i, D^a, M, b_μ through their equations of motion, which is easy as well. At the end of the computation, it turns out that the Lagrangian is not canonically normalized. In order to have a canonically normalized Lagrangian, we must perform a dilatation of the vierbein, followed by a gravitino shift [151], which amount to a re-definition of all the fields and can be interpreted as a superconformal (or Howe-Tucker) transformation. This is certainly the most difficult step.

其中 m_p 是普朗克质量。为了更清晰地展开，我们需要将所有场按 Θ 变量展开。由于所有超场都包含大量分量，这个计算过程冗长，但本质上并不复杂。展开完成后，我们还需要通过运动方程消去辅助场 F^i, D^a, M, b_μ ，这一步同样简单。计算结束后会发现拉格朗日量并非正则归一化的。为了得到正则归一化的拉格朗日量，我们必须先对四标架做伸缩变换，再做引力微子平移 [151]，这等价于重新定义所有场，也可以解释为超共形 (即 Howe-Tucker) 变换。这无疑是整个过程中最困难的一步。

Due to the technical difficulties associated with the field re-definition, alternative methods have been proposed in order to compute the final Lagrangian. The first method [14] is geometrical in nature, enlarging the usual superspace to a $U(1)$ - superspace. The second approach is based on superconformal methods. In Poincaré supergravity, the structure group is the Poincaré supergroup. The conformal methods [66-68, 106, 107, 147] are based on an enlargement of the structure group to the superconformal group. The book of Freedman and van Proeyen [69] is devoted to conformal methods in supergravity.

由于场重新定义带来了诸多技术困难，人们已经提出了替代方法来计算最终的拉格朗日量。第一种方法 [14] 本质上是几何方法，将通常的超空间扩充为 $U(1)$ 超空间。第二种方法基于超共形方法。在庞加莱超引力中，结构群是庞加莱超群。共形方法 [66-68, 106, 107, 147] 的核心是将结构群扩充为超共形群。Freedman 与 van Proeyen 的著作 [69] 专门介绍了超引力中的共形方法。

The final Lagrangian contains the Einstein-Hilbert action for general relativity, the Rarita-Schwinger Lagrangian for the spin 3/2-gravitino, the kinetic terms of the matter sector (spin $0, 1/2$), and the kinetic part of the Yang-Mills sector (spin $1, 1/2$) . All these fields are naturally coupled and invariant under supergravity transformations, but also under many other transformations, such as symmetries of the underlying Kähler manifold (the complex manifold where chiral fields are living), etc. Many interacting terms are also generated, such as four-fermion interactions. The scalar potential takes the form (we denote $W_{\ i} = \partial_i W, K_i = \partial_i K$ etc., $K^{i^*}{}_i = \partial^{i^*}\partial_i K$ the Kähler metric of the Kähler manifold and $K^i{}_{i^*}$ the inverse Kähler metric) (In (8), we have omitted the usual D -terms (see section "Supergravity Breaking").):

最终拉格朗日量包含广义相对论的爱因斯坦-希尔伯特作用量、自旋 3/2 引力微子的拉里塔-施温格拉格朗日量、物质部分 (自旋 $0, 1/2$) 的动能项，以及杨-米尔斯部分 (自旋 $1, 1/2$)) 的动能项。所有这些场自然耦合，且在超引力变换下不变，同时在许多其他变换下也保持不变，例如 underlying 凯勒流形 (手征场所处的复流形) 的对称性等。还会生成许多相互作用项，例如四费米子相互作用。标量势形式如下 (我们记 $W_{\ i} = \partial_i W, K_i = \partial_i K$ 等， $K^{i^*}{}_i = \partial^{i^*}\partial_i K$ 为凯勒流形的凯勒度量， $K^i{}_{i^*}$ 为逆凯勒度量)(式 (8) 中我们省略了常见的 D 项，参见“超引力破缺”一节):

$$V = e^{\frac{1}{m_p^2}K(\varphi,\varphi^\dagger)}\left(\mathcal{D}_i W \mathcal{D}^{i^*} W^\star K^i{}_{i^*} - \frac{3}{m_p^2}|W|^2\right), \tag{8}$$

where $\mathcal{D}_i W = W_{\ i} + 1/m_p^2 K_i W$ with a similar expression for $\overline{\mathcal{D}}^{i^*} W^\star$ are covariant derivatives with respect to the Kähler manifold structure. It is remarkable that the structure of the final Lagrangian emerges from strictly geometrical and algebraic principles that are related to the properties of the curved superspace and its associated supergravity transformations.

其中 $\mathcal{D}_i W = W_{\ i} + 1/m_p^2 K_i W$ 和形式类似的 $\overline{\mathcal{D}}^{i^*} W^\star$ 是关于凯勒流形结构的协变导数。值得注意的是，最终拉格朗日量的结构完全来源于几何与代数原理，这些原理与弯曲超空间及其相关超引力变换的性质相关。

If we now study the low-energy limit of (7), namely, when $m_p \to +\infty$, the part of the Lagrangian involving the Kähler potential becomes

如果我们现在研究 (7) 的低能极限，即当 $m_p \to +\infty$ 时，拉格朗日量中涉及凯勒势的部分变为

$$\mathcal{L} = -3m_p^2 \int \mathrm{d}^2\Theta \mathcal{E}\mathcal{R} - \frac{1}{8}\int \mathrm{d}^2\Theta\mathcal{E}\left(\overline{\mathcal{D}}\cdot\overline{\mathcal{D}} - 8\mathcal{R}\right) K\left(\Phi, \Phi^\dagger e^{-2gV}\right) + \text{ h.c. } + \mathcal{O}\left(\frac{1}{m_p^2}\right).$$

The first piece reduces to [151]

第一项化简为 [151]

$$\mathcal{L}_{\text{ pure SUGRA }} = em_p^2\left(\frac{1}{2}R + \frac{1}{4}\varepsilon^{\mu\upsilon\rho\sigma}\left[\psi_\mu\sigma_\sigma\overline{\psi}_{\upsilon\rho} - \overline{\psi}_\mu\overline{\sigma}_\sigma\psi_{\upsilon\rho}\right] - \frac{1}{3}\left[MM^\star + b_\mu b^\mu\right]\right)$$

where R is the scalar curvature, e is the determinant of the vierbein, $\psi_{\mu\nu}$ is the field strength of the gravitino, and $\varepsilon^{\mu\upsilon\rho\sigma}$ is the Levi-Civita symbol. This is the Lagrangian obtained in [31, 70] including the auxiliary fields [60, 143] and describing pure supergravity.

其中 R 是标量曲率，e 是四标架的行列式，$\psi_{\mu\nu}$ 是引力微子的场强，$\varepsilon^{\mu\upsilon\rho\sigma}$ 是列维-奇维塔符号。这就是 [31, 70] 中得到的拉格朗日量，它包含辅助场 [60, 143]，描述纯超引力。

In the limit $m_p \to +\infty$, the projector $\left(\overline{\mathcal{D}}\cdot\overline{\mathcal{D}} - 8\mathcal{R}\right)$ becomes $\bar{D}\cdot\bar{D}$ with $\bar{D}$ (and D) the usual covariant derivatives in supersymmetry, $\mathcal{W}_\alpha$ becomes W_α the spinor field strength in supersymmetry [73, 151] , and Θ reduce to θ the left-handed Grassmann variables considered in supersymmetry. If we now restrict ourselves to a renormalizable theory, the basic functions reduce to

在 $m_p \to +\infty$ 极限下，投影算子 $\left(\overline{\mathcal{D}}\cdot\overline{\mathcal{D}} - 8\mathcal{R}\right)$ 变为 $\bar{D}\cdot\bar{D}$ ，其中 $\bar{D}$ (以及 D) 是超对称中的常规协变导数，$\mathcal{W}_\alpha$ 变为超对称中的旋量场强 W_α [73, 151] ，Θ 约化为超对称中考虑的左手格拉斯曼变量 θ 。如果我们限定讨论可重整化理论，基本函数简化为

$$K\left(\Phi, \Phi^\dagger\right) = \Phi_i^\dagger\Phi^i, h_{ab}\left(\Phi\right) = \delta_{ab}, W\left(\Phi\right) = \alpha_i\Phi^i + \frac{1}{2}m_{ij}\Phi^i\Phi^j + \frac{1}{6}\lambda_{ijk}\Phi^i\Phi^j\Phi^k.$$

Therefore, omitting Planck-suppressed terms, we obtain in the limit $m_p \to +\infty$

因此，省略普朗克压低的项后，我们在 $m_p \to +\infty$ 极限下得到

$$\mathcal{L}_{\text{ SUSY }} = \int \mathrm{d}^2\theta\left\{-\frac{1}{8}\bar{D}\cdot\bar{D}\left(\Phi^\dagger e^{-2gV}\Phi\right) + W + \frac{1}{16g^2}\delta_{ab}W^{a\alpha}W_\alpha^b\right\} + \text{ h.c.} \tag{9}$$

which is the standard action in supersymmetry expressed in a chiral form. After expanding the fields, we obtain

这就是手征形式表示的超对称标准作用量。展开场量后，我们得到

$$
\begin{aligned}
\mathcal{L}_{\text{SUSY}} = & -\frac{1}{4}\delta_{ab}F^{a\mu\nu}F^{b}_{\mu\nu} - \frac{i}{2}\delta_{ab}\left(\lambda^a\sigma^\mu D_\mu\overline{\lambda}^b - D_\mu\lambda^a\sigma^\mu\overline{\lambda}_a\right) + \frac{1}{2}\delta_{ab}D^aD^b \\
& + D_\mu\varphi^\dagger D^\mu\varphi - \frac{i}{2}\left(\chi\sigma^\mu D_\mu\overline{\chi} - D_\mu\chi\sigma^\mu\overline{\chi}\right) + F^\dagger F \\
& - gD^a\varphi^\dagger T_a\varphi - i\sqrt{2}g\overline{\lambda}^a\cdot\overline{\chi}T_a\varphi + i\sqrt{2}g\varphi^\dagger T_a\chi\cdot\lambda^a \\
& - \left(m_{ij}\left(\varphi^iF^j + \frac{1}{2}\chi^i\cdot\chi^j\right) - \frac{1}{2}\lambda_{ijk}\left(\varphi^i\varphi^jF^k + \varphi^i\chi^j\cdot\chi^k\right) + \text{ h.c. }\right),
\end{aligned}
\tag{10}
$$

where $F^a_{\mu\nu} = \partial_\mu v^a_\nu - \partial_\nu v^a_\mu - gf_{bc}{}^a v^b_\mu v^c_\nu$ and $D_\mu\lambda^a = \partial_\mu\lambda^a - gf_{bc}{}^a v^b_\mu\lambda^c$ are, respectively, the field strength tensor and the covariant derivative in the Yang-Mills sector and $D_\mu\varphi^i = \partial_\mu\varphi^i + igv^a_\mu(T_a\varphi)^i$ and $D_\mu\chi^i = \partial_\mu\chi^i + igv^a_\mu(T_a\chi)^i$ are the covariant derivatives in the matter sector. Finally, $\sigma^\mu_{\alpha\dot\alpha} = (\sigma^0, \sigma^i)$ and $\overline{\sigma}^{\mu\dot\alpha\alpha} = (\sigma^0, -\sigma^i)$ where $\sigma^i, i = 1,2,3$ are the usual Pauli matrices and σ^0 is the two-by-two identity matrix. For completeness, we remind our conventions for the Minkowski metric and Levi-Civita symbol $\eta_{\mu\nu} = \text{diag}(1,-1,-1,-1)$ and $\varepsilon_{0123} = 1$. If we eliminate the auxiliary fields through their equations of motion, we get

其中 $F^a_{\mu\nu} = \partial_\mu v^a_\nu - \partial_\nu v^a_\mu - gf_{bc}{}^a v^b_\mu v^c_\nu$ 和 $D_\mu\lambda^a = \partial_\mu\lambda^a - gf_{bc}{}^a v^b_\mu\lambda^c$ 分别是杨-米尔斯部分的场强张量和协变导数，$D_\mu\varphi^i = \partial_\mu\varphi^i + igv^a_\mu(T_a\varphi)^i$ 和 $D_\mu\chi^i = \partial_\mu\chi^i + igv^a_\mu(T_a\chi)^i$ 是物质部分的协变导数。最后，得到 $\sigma^\mu_{\alpha\dot\alpha} = (\sigma^0, \sigma^i)$ 和 $\overline{\sigma}^{\mu\dot\alpha\alpha} = (\sigma^0, -\sigma^i)$，其中 $\sigma^i, i = 1,2,3$ 是常规泡利矩阵，σ^0 是二阶单位矩阵。为完整起见，此处说明我们对闵氏度规和列维-奇维塔符号的约定为 $\eta_{\mu\nu} = \text{diag}(1,-1,-1,-1)$ 和 $\varepsilon_{0123} = 1$ 。若我们通过运动方程消去辅助场，可得

$$
F^\dagger_i = \partial_i W \equiv W_{\,i} \text{ and } D_a = g\varphi^\dagger T_a\varphi \tag{11}
$$

and obtain the scalar potential, with its F - and D -term parts (we denote $D_a = \delta_{ab}D^b, T_a = \delta_{ab}T^b$, etc.)

并得到标量势，其包含 F 项和 D 项两部分 (我们记为 $D_a = \delta_{ab}D^b, T_a = \delta_{ab}T^b$ 等形式)

$$
\begin{aligned}
V = V_F + V_D = F^\dagger_i F^i + \frac{1}{2}D_aD^a = W_{\,i}W^{\star i} + \frac{1}{2}g^2\left(\varphi^\dagger T_a\varphi\right)\left(\varphi^\dagger T^a\varphi\right) = \left(\alpha_i + m_{ij}\varphi^i\right. \\
\left. + \frac{1}{2}\lambda_{ijk}\varphi^j\varphi^k\right)\left(\alpha^{*i} + m^{*i\ell}\varphi^\dagger_\ell + \frac{1}{2}\lambda^{*i\ell m}\varphi^\dagger_\ell\varphi^\dagger_m\right) + \frac{1}{2}g^2\delta^{ab}\left(\varphi^\dagger T_a\varphi\right)\left(\varphi^\dagger T_b\varphi\right)
\end{aligned}
$$

(12)

and the Yukawa interactions between spinors and scalar fields

以及旋量与标量场之间的汤川相互作用

$$
\mathcal{L}_{\text{ Yuk }} = -\frac{1}{2}W_{\,ij}\chi^i\cdot\chi^j + \text{ h.c. } = -\lambda_{ijk}\varphi^i\chi^j\cdot\chi^k + \text{ h.c.} \tag{13}
$$

For explicit computations in the supersymmetry context, the reader may refer to the book by one of the authors [73]. We further stress that the supergravity action is derived with all details in [45, 148]. As a complement to this section, one may consult [29] in the same volume, where simple supergravity in components is presented, [110] which provides a superspace formulation of supergravity, or [111], related to supergravity-matter couplings in projective superspace.

关于超对称框架下的具体计算，读者可参考其中一位作者的著作 [73]。我们还要强调，超引力作用量的完整推导可见文献 [45, 148]。作为本节的补充，读者可参考同卷中的文献 [29]，其中给出了简单超引力的分量形式；文献 [110] 给出了超引力的超空间表述；文献 [111] 则与投影超空间中的超引力-物质耦合相关。

Supergravity Breaking

超引力破缺

In the previous section, we have described the basic steps allowing to construct a Lagrangian coupling matter and Yang-Mills theory to supergravity. Its limit for $m_\mathrm{p} \to \infty$, i.e., the supersymmetric theory, has been simultaneously obtained. This derivation opens the path to applications of the supergravity/supersymmetry formalism to particle physics. At the center of any such application is the gain in regularity of a QFT protected by supersymmetry, taking the form of non-renormalization theorems [87]. Corresponding particle physics models may then remain perturbative over the ”Grand Desert” separating the electroweak from the Planck scales, thus preserving the general structure of the studied model over a huge hierarchy of scales: this is the type of applications in which we specialize below. However, this same advantage of regular quantum corrections also finds use in the study of the general properties of Yang-Mills theories at the quantum level (”integrability”) [39, 52, 94]. In particular, implementing supersymmetry/supergravity in particle physics enables the protection of the mass of scalar fields from quantum corrections: a massless scalar field remains massless at any order of perturbation theory. We will argue in the following paragraph that supersymmetry needs to be broken for phenomenological applications; still, this breaking can be enforced in such a ”soft” way that this nice property under renormalization is preserved. In this fashion, softly broken supersymmetry provides a technical solution to the hierarchy problem [32,108,132,156,158]. Nevertheless, supersymmetry breaking terms should remain comparable to the electroweak scale so that supersymmetry actively protects the Higgs mass from comparatively low scales: observable effects at colliders are then expected.

在上一节中，我们介绍了将物质与杨-米尔斯理论耦合到超引力中构造拉格朗日量的基本步骤，同时也得到了其在 $m_\mathrm{p} \to \infty$ 下的极限，即超对称理论。这一推导为超引力/超对称形式体系应用于粒子物理开辟了道路。所有此类应用的核心是，超对称保护的量子场论具备更高的正则性，具体体现为非重整化定理 [87]。相应的粒子物理模型可以在分隔电弱标度与普朗克标度的“大荒漠”中保持微扰性，从而在巨大的标度层级下保留研究模型的整体结构: 我们下文将专门讨论这类应用。此外，量子修正具备正则性这一优势，也可用于研究量子层面杨-米尔斯理论的一般性质 (“可积性”)[39, 52, 94]。具体而言，在粒子物理中引入超对称/超引力，可以保护标量场质量不被量子修正: 零质量标量场在任意阶微扰论中都保持零质量。我们将在下文说明，超对称必须破缺才能符合唯象应用；不过，破缺可以以“软”方式实现，使得重整化下的优良性质得以保留。通过这种方式，软破缺超对称为层级问题提供了一种技术解决方案 [32,108,132,156,158]。尽管如此，超对称破缺项的大小需要与电弱标度相当，这样超对称才能在较低标度下持续保护希格斯质量，因此我们有望对撞机上观测到可观测效应。

Due to the fermionic nature of the generators of supersymmetry/supergravity transformations, a multiplet contains both bosonic and fermionic degrees of freedom, which find a natural embedding in the chiral or the vector superfields of section ”How to Build an Action in Supergravity.” In particular, if one wants to construct a supersymmetric version of the Glashow-Weinberg-Salam model, each standard particle must be associated with a supersymmetric partner: a scalar to each fermion and a fermion to each boson (gauge and Higgs). Noticing at this point that the operator $P_\mu P^\mu$ is a Casimir operator, particles and their superpartners are necessarily degenerate in mass if supersymmetry is unbroken. More specifically, this means that if supergravity/supersymmetry were an exact symmetry of nature, a massless neutral fermion (associated with the photon) or a light charged scalar (associated with the electron) should have been observed by now, which is obviously not the case. Thus, supersymmetry/supergravity must be broken at the scale of current particle physics experiments.

> 由于超对称/超引力变换的生成子具有费米子性质, 一个多重态同时包含玻色子和费米子自由度, 二者可以自然嵌入我们在“如何在超引力中构造作用量”一节介绍的手征或矢量超场中。具体而言, 如果要构造格拉肖-温伯格-萨拉姆模型的超对称版本，每个标准模型粒子都必须对应一个超对称伙伴: 每个费米子对应一个标量，每个玻色子 (规范玻色子和希格斯玻色子) 对应一个费米子。至此我们可以看到，算符 $P_\mu P^\mu$ 是一个卡西米尔算符，如果超对称未破缺，粒子与其超伙伴一定质量简并。具体来说，这意味着如果超引力/超对称是自然界的精确对称性，那么我们现在应该已经观测到零质量中性费米子 (对应光子) 和轻带电标量 (对应电子)，但显然事实并非如此。因此，超对称/超引力必须在当前粒子物理实验的能标下发生破缺。

Mechanisms of Supersymmetry and Supergravity Breaking

超对称与超引力破缺的机制

We first recall the classical mechanisms achieving a (global) breaking of super-symmetry. One way to induce a spontaneous breaking consists in designing a field configuration such that the equations of motions for the auxiliary fields F or D are incompatible with the trivial solution, i.e., at least one of the auxiliary fields develops a vacuum expectation value (vev). Correspondingly, the scalar potential (13) becomes strictly positive at the minimum in a spontaneously broken supersymmetric theory. The first mechanism (O’Raifeartaigh) of supersymmetry breaking achieves $F \neq 0$ and involves at least three chiral superfields [127]. The second mechanism (Fayet-Iliopoulos) induces $D \neq 0$ through a vector superfield associated with a $U(1)$ -gauge symmetry, allowing for a Fayet-Iliopoulos term in the Lagrangian [54,59]. Both mechanisms fail to produce a spectrum compatible with the experimental situation, as at least some of the ”exotic” particles would be lighter than their standard partners. Thus, the Standard Model cannot be straightforwardly embedded in a spontaneously broken supersymmetric framework. For some details related to these two mechanisms, see, e.g., [73].

我们首先回顾实现 (整体) 超对称破缺的经典机制。诱导自发破缺的一种方式是构造这样的场组态: 辅助场 F 或 D 的运动方程与平庸解不相容，即至少一个辅助场获得真空期望值 (vev)。相应地，在自发破缺的超对称理论中，标量势 (13) 在极小值处严格为正。第一个超对称破缺机制 (O'Raifeartaigh) 实现了 $F \neq 0$ ，且至少包含三个手征超场 [127]。第二个机制 (Fayet-Iliopoulos) 通过与 $U(1)$ 规范对称性关联的矢量超场诱导出 $D \neq 0$ ，使得拉格朗日中可以存在 Fayet-Iliopoulos 项 [54,59]。两种机制都无法得到与实验情况相容的能谱，因为至少部分“奇异”粒子会比它们的标准模型伴粒子更轻。因此，标准模型无法直接嵌入自发破缺的超对称框架中。关于这两种机制的更多细节，参见例如文献 [73]。

However, this picture could play a rôle if a less simplistic structure is introduced: supersymmetry is spontaneously broken in a hidden sector by one of the two mechanisms described above; then this breaking is transmitted to an observable (or matter) sector, which contains a supersymmetric extension of the Standard Model, by the interaction connecting both sectors. The nature of this interaction determines three classes of mediation models. In the first type, known as gravity-mediated [3, 11, 22, 46, 97, 124, 126, 128] , the supersymmetry breaking is communicated to the observable sector via gravitational interactions. In the second type, called gauge mediation, supersymmetry is broken by a singlet field (called the spurion), which couples (via the superpotential) to another field, the messenger, belonging to a non-trivial representation of the gauge group. Quantum loop corrections involving the messenger then break supersymmetry in the observable sector [2, 5, 30, 33 − 38, 57, 79, 120, 123] . In the third type, called anomaly mediation, the introduction of compensating fields having a conformal (super-Weyl) anomaly induces supersym-metry breaking by purely quantum effects [5, 9, 130] . Concerning general properties of broken supergravity, we refer the reader to [59, 99, 156, 157]. Here, we will restrict ourselves to the specific case of gravity-induced supersymmetry breaking in section "Gravity-Induced Supersymmetry Breaking."

不过, 如果引入不那么简单的结构, 这一图像仍可以成立: 超对称由上述两种机制之一在隐藏区自发破缺, 随后破缺通过连接两个区的相互作用传递到包含标准模型超对称推广的可观测区 (或物质区)。这种相互作用的性质决定了三类传播模型。第一类称为引力传播模型 [3, 11, 22, 46, 97, 124, 126, 128] ，超对称破缺通过引力相互作用传递到可观测区。第二类称为规范传播模型: 超对称由一个单态场 (称为赝标场) 破缺，该场 (通过超势) 耦合到另一个属于规范群非平凡表示的场，即传播子。包含传播子的量子圈修正随后在可观测区破缺超对称 [2, 5, 30, 33 − 38, 57, 79, 120, 123] 。第三类称为反常传播模型: 引入具有共形 (超外尔) 反常的补偿场，可仅通过量子效应诱导超对称破缺 [5, 9, 130] 。关于破缺超引力的一般性质，我们建议读者参阅文献 [59, 99, 156, 157]。本文将在“引力诱导超对称破缺”一节专门讨论引力诱导超对称破缺的具体情况。

As seen in (8), the scalar potential in supergravity, hence its minimum, is no longer positive (as in global supersymmetry). This turns out to be an advantage, since, supergravity being a theory of gravitation, one expects that the cosmological constant, hence the potential at the minimum, vanishes. Furthermore, as will be established in section "The Goldstino," after supergravity breaking, the gravitino becomes massive. The assumption of a vanishing cosmological constant, though phenomenologically motivated (as we just stated), appears as a fine-tuning problem in general, except in theories called no-scale supergravity (see section "No-Scale Supergravity") where geometrical properties of the Kähler manifold lead to a vanishing potential. This condition of a vanishing cosmological constant then promotes the gravitino mass to the status of an order parameter.

正如我们从 (8) 中看到的，超引力中的标量势 (以及它的极小值) 不再像整体超对称中那样为正。这实际上是一个优势: 因为超引力是引力理论，我们预期宇宙学常数 (即势在极小值处的值) 为零。此外，我们将在“戈德斯提诺”一节说明，超引力破缺后，引力微子获得质量。尽管零宇宙学常数的假设符合唯象动机 (正如我们刚才所述)，但一般来说它是一个精细调节问题，只有无标度超引力理论除外 (参见“无标度超引力”一节)，这类理论中凯勒流形的几何性质使得势自动为零。零宇宙学常数的条件继而让引力微子质量成为序参量。

The Goldstino

戈德斯提诺

Let us first consider a supersymmetric theory with the Lagrangian (10) and suppose that it is broken by some mechanism among those briefly described in section ”Mechanisms of Supersymmetry and Supergravity Breaking,” i.e., that some of the auxiliary fields F^i and D^a and some of the scalar fields φ^i develop a vev (We do not consider fermion condensates in this analysis.). We thus obtain the fermion mass matrix (see (10) and (13))

我们首先来考虑一个满足拉格朗日量 (10) 的超对称理论，假设该理论通过「超对称与超引力破缺的机制」一节中简要介绍的某一机制发生破缺，即部分辅助场 F^i 和 D^a 、部分标量场 φ^i 获得真空期望值 (我们本次分析不讨论费米子凝聚)。由此我们可以得到费米子质量矩阵 (见 (10) 和 (13))

$$\mathcal{L}_{\text{mass}} = -\frac{1}{2}\left(\chi^i i\sqrt{2}\lambda^a\right)\mathcal{M}_F\begin{pmatrix}\chi^j\\ i\sqrt{2}\lambda^b\end{pmatrix}, \mathcal{M}_F \equiv \begin{pmatrix}\langle W_{ij}\rangle_i & -g\langle\varphi^\dagger T_b\rangle_i\\ -g\langle\varphi^\dagger T_a\rangle_j & 0\end{pmatrix}.$$

Using the minimization of the potential (13), the gauge invariance of the superpo-tential, and (11), we have

利用势的极小化 (13)、超势的规范不变性以及 (11)，我们可得

$$\begin{cases}\partial_i\langle V\rangle = \langle W_{ij}\rangle\langle F^i\rangle + g\langle\varphi^\dagger T_a\rangle_i\langle D^a\rangle = 0\\ \delta_a W^\star = W^{\star i}\delta_a\varphi_i^\dagger = -F^i(\varphi^\dagger T_a)_i = 0\end{cases} \Rightarrow \mathcal{M}_F\begin{pmatrix}\langle F^i\rangle\\ -\langle D^a\rangle\end{pmatrix} = 0, \tag{14}$$

and thus, a massless field emerges, called the goldstino or the Goldstone fermion:

因此会产生一个无质量场，称为戈德斯蒂诺即戈德斯通费米子:

$$\Psi_G = \frac{1}{\sqrt{2}}\left(\left\langle F_i^\dagger\right\rangle\chi^i - \frac{i}{\sqrt{2}}\langle D_a\rangle\lambda^a\right) = \frac{1}{\sqrt{2}}\left\langle F_i^\dagger\right\rangle\chi^i - \frac{i}{2}\langle D_a\rangle\lambda^a. \tag{15}$$

It is the analogue for broken supersymmetry of the Golstone boson in broken gauge theories [98, 133]. However, since such a massless particle was not observed, again, the absence of a massless fermion in nature pleads against a spontaneous breaking of global supersymmetry.

它是破缺规范理论中戈德斯通玻色子在破缺超对称中的对应粒子 [98, 133]。然而，由于这种零质量粒子从未被观测到，自然界中不存在零质量费米子这一点同样不支持整体超对称的自发破缺。

The inconvenience of a massless goldstino is lifted in supergravity by a phenomenon comparable to the Brout-Englert-Higgs mechanism for the electroweak symmetry. We derive it below, assuming for simplicity, and since this does not modify the conclusion, a canonical Kähler potential, and a trivial gauge kinetic function as in (9). The interaction terms for fermions are the same as in super-symmetry mutatis mutandis the term W_{ij}, which becomes $\mathcal{D}_i\mathcal{D}_j W$ with $\mathcal{D}_i$, the covariant derivative with respect to the Kähler manifold (for notations see [148]), and the presence of an exponential factor:

在超引力中，无质量戈德斯蒂诺的缺陷可以通过一种可类比电弱对称的布劳特-恩格勒-希格斯机制的现象消除。我们下文将推导这一过程: 为简单起见，且由于该设定不改变结论，我们假设存在一个典范凯勒势，以及如式 (9) 所示的平凡规范动力学函数。费米子的相互作用项与超对称中的形式相同，只需对项 W_{ij} 作相应变更: 该项变为 $\mathcal{D}_i\mathcal{D}_j W$，其中包含 $\mathcal{D}_i$ ——即关于凯勒流形的协变导数 (记号参见文献 [148])，还多出一个指数因子:

$$e^{-1}\mathcal{L}_{\text{ferm}} = -\frac{1}{2}e^{\frac{\varphi^\dagger\varphi}{m_P^2}}\mathcal{D}_i\mathcal{D}_j W\chi^i\cdot\chi^j - i\sqrt{2}g\overline{\lambda}^a\cdot\overline{\chi}T_a\varphi + \text{ h.c.} \tag{16}$$

The scalar potential consists of (8) (which is in fact the F-term and the N-term of the potential. The auxiliary field N is related in a simple way to the field M after a change of variables.) and the usual D-term of supersymmetry. After solving the equation of motion for the auxiliary fields, one obtains (see, e.g., [148])

标量势由 (8)(其实际上是势的 F 项和 N 项。经过变量替换后，辅助场 N 与场 M 存在简单关联) 以及超对称中常见的 D 项构成。求解辅助场的运动方程后，可得 (例如参见 [148])

$$\langle F^i\rangle = e^{\frac{1}{2}\frac{\langle\varphi^\dagger\varphi\rangle}{m_P^2}}\langle\mathcal{D}^i W^\star\rangle, \langle D^a\rangle = g\langle\varphi^\dagger T_a\varphi\rangle, \langle N\rangle = -3e^{\frac{1}{2}\frac{\langle\varphi^\dagger\varphi\rangle}{m_P^2}}\langle W\rangle$$

and the goldstino of (15) emerges as a massless field, as in supersymmetry.

且 (15) 中的戈德斯蒂诺作为无质量场出现，这与超对称中的情况一致。

However, under a supergravity transformation, we have

然而，在超引力变换下，我们可得

$$\delta_\varepsilon\chi^i = \sqrt{2}\varepsilon F^i + \cdots, \delta_\varepsilon\lambda^a = i\varepsilon D^a + \cdots$$

where the dots indicate terms that are irrelevant for our analysis. In particular, the goldstino transforms as

其中省略号表示对我们的分析无关紧要的项。金戈斯蒂诺 (Goldstino) 的变换形式为

$$\delta_\varepsilon\Psi_G = \varepsilon\left(\langle F_i^\dagger F^i\rangle + \frac{1}{2}\langle D_a D^a\rangle\right) + \cdots \tag{17}$$

which involves the F- and D-parts of the scalar potential. As the prefactor of the symmetry parameter ε is non-zero for broken supergravity, a local choice of ε allows to purge the Lagrangian from the goldstino field (similar to the Goldstone boson in broken gauge theory) [24, 25, 27] . In this process, the gravitino acquires a mass

它涉及标量势的 F 部分和 D 部分。对于破缺超引力，对称参数 ε 的前因子非零，因此局域选取 ε 可将金微子从拉格朗日量中消去 (类似破缺规范理论中的戈德斯通玻色子) [24, 25, 27] 。在这一过程中，引力微子获得质量

$$m_{\frac{3}{2}} = \frac{1}{m_p^2} \left\langle W e^{\frac{1}{2}\frac{K}{m_p^2}} \right\rangle. \tag{18}$$

The irrelevance of the goldstino as a dynamical field can also be exhibited by an explicit diagonalization of the mass matrix, in which a field re-definition makes it disappear [69, 148, 151]. Depending on the mediation mechanism, the gravitino mass can take very different values. For instance, in gauge mediation, it is typically very light, because it is m_p -suppressed (and it can be the lightest supersymmetric particle). We shall see in the next subsection that in gravity mediation, it determines the supersymmetry breaking scale (under the assumption of a vanishing cosmological constant) and thus remains comparable to the electroweak scale, i.e., $\sim 100\text{GeV} - 1\text{TeV}$.

金戈斯蒂诺作为动力学场的非相关性也可以通过质量矩阵的显式对角化展示，在该过程中重新定义场即可让它消失 [69, 148, 151]。根据不同的传递机制，引力微子质量可以取差异极大的数值。例如，在规范传递中，它通常非常轻，因为它受 m_p 压低 (且它可以是最轻超对称粒子)。我们将在下一小节看到，在引力传递中，它决定了超对称破缺的能标 (在宇宙学常数为零的假设下)，因此其大小仍然与电弱能标相当，即 $\sim 100\text{GeV} - 1\text{TeV}$ 。

Gravity-Induced Supersymmetry Breaking

引力诱导超对称破缺

In this section, we focus on gravity-induced supersymmetry breaking. The chiral superfields are denoted as $X^I = \left(Z^i, \Phi^a\right)$, $i = 1, \ldots, m$, $a = 1, \ldots, n$ where Z (resp. Φ) are the superfields in the hidden (resp. observable) sector (the corresponding scalar fields are denoted as $x^I = \left(z^i, \varphi^a\right)$). We do not specify the observable sector here, which corresponds to a supersymmetric extension of the Standard Model (see section ”Supergravity and Supersymmetry in Particle

本节我们聚焦引力诱导超对称破缺。手征超场记为 $X^I = \left(Z^i, \Phi^a\right)$，$i = 1, \ldots, m$，$a = 1, \ldots, n$ ，其中 Z (对应 Φ) 是隐藏 (对应可见) sector 的超场，对应的标量场记为 $x^I = \left(z^i, \varphi^a\right)$ 。我们在此不具体说明可见 sector，它对应标准模型的超对称推广 (参见“粒子”部分“超引力与超对称”

Physics”). Of course, the model also contains vector superfields associated with the Yang-Mills interactions, but these fields are irrelevant for the derivation of the supersymmetry breaking terms in the observable sector, which we perform in this subsection. In addition, we assume that supergravity is broken in the hidden sector by an O’Raifeartaigh mechanism. More specifically, we suppose that some of the fields of the hidden sector develop a vev of the order of the Planck mass, while vevs for the fields in the observable sector would be of the order of the electroweak or GUT scales and thus subleading.

物理”)。当然，模型也包含与杨-米尔斯相互作用关联的矢量超场，但在本小节推导可见 sector 的超对称破缺项时，这些场无关紧要。此外我们假设，超引力在隐藏 sector 由 O'Raifeartaigh 机制破缺。更具体地说，我们假设隐藏 sector 的部分场获得量级为普朗克质量的真空期望值，而可见 sector 场的真空期望值量级为电弱标度或大统一标度，因此是次 Leading 项。

Before expanding the scalar potential (8) in the vicinity of its minimum for $m_p \to \infty$ (now in the presence of vevs of order m_p , contrary to the discussion of section "How to Build an Action in Supergravity" with unbroken supergravity), it is necessary to specify the behavior of the terms connecting hidden and observable sectors in this limit: these are assumed to remain finite when $m_p \to \infty$, which considerably restricts the possible form of the Kähler potential and superpotential, as was demonstrated by Soni and Weldon in [140]. The form retained by Soni and Weldon leads to a soft breaking of supersymmetry in the observable sector for the limit $m_p \to \infty$, as phenomenologically desirable in traditional applications of supersymmetry to particle physics, so that only UV divergences of logarithmic type appear in quantum corrections, thus protecting electroweak scalar masses against corrections from the UV spectrum (hierarchy problem). Nevertheless, we note that supergravity breaking admits solutions beyond those of Soni and Weldon, with in particular possible hard breaking terms. Models of this type were analyzed in [117] and are not deprived of phenomenological qualities. Nevertheless, we will focus below on the more commonly studied case of soft breaking.

在标量势 (8) 在其极小值附近对 $m_p \to \infty$ 做展开之前 (此时存在量级为 m_p 的真空期望值，和“超引力中如何构造作用量”一节未破缺超引力的讨论不同)，需要先确定该极限下连接隐藏 sector 和可见 sector 的项的行为: 一般假设当 $m_p \to \infty$ 时这些项保持有限，这就像 Soni 和 Weldon 在文献 [140] 中证明的那样，对 Kähler 势和超势的可能形式做出了相当大的限制。在极限 $m_p \to \infty$ 下，Soni 和 Weldon 保留的形式会在可见 sector 产生软超对称破缺，这符合超对称应用于粒子物理传统场景中的唯象要求: 量子修正中仅会出现对数型紫外发散，从而保护电弱标量质量不受紫外能谱修正 (层级问题)。不过我们需要指出，超引力破缺还存在 Soni 和 Weldon 工作之外的解，其中尤其可能包含硬破缺项。这类模型已在文献 [117] 中分析，且并非没有唯象优势。但下文我们仍会聚焦更常用的软破缺情形展开讨论。

In [15, 78] , the Kähler potential and superpotential were chosen as

在 [15, 78] 中，Kähler 势和超势取为

$$K\left(Z, Z^{\dagger}, \Phi, \Phi^{\dagger}\right)=\widehat{K}\left(Z, Z^{\dagger}\right)+\Phi_{a^*}^{\dagger} \Lambda^{a^*}{}_{a}\left(Z, Z^{\dagger}\right) \Phi^{a}+\left(\Gamma_{k}\left(Z, Z^{\dagger}\right) g_{2}^{k}(\Phi)+\text { h.c. }\right)$$

$$W(Z, \Phi)=\widehat{W}(Z)+W_{k}(Z)\, g_{3}^{k}(\Phi) .$$

Remembering the relative mass dimensions of these quantities, it is further assumed that the realization of the Kähler potential in the hidden (resp. observable) sector scales like (This assumption is a consequence of the results of Soni and Weldon [143].) $\widehat{K} \sim m_p^2$ (resp. M^2), while the superpotential is of order $\widehat{W} \sim M m_p^2$ (resp. M^3), with $M \ll m_p$. There are a priori no restrictions on the form of the functions g_2^k and g_3^k and the general form of the scalar potential after supergravity breaking can be found in [15, 78] .

记住这些量的质量量纲关系后，进一步假设: 隐藏 (对应可见) sector Kähler 势的标度为 (该假设是 Soni 和 Weldon[143] 结果的推论): $\widehat{K} \sim m_p^2$ (对应 M^2)，而超势的量级为 $\widehat{W} \sim M m_p^2$ (对应 M^3)，满足 $M \ll m_p$ 。原则上对函数 g_2^k 和 g_3^k 的形式没有限制，超引力破缺后标量势的一般形式可在 [15, 78] 中找到。

For the current analysis, however, we assume a renormalizable theory in the observable sector and specify

但出于当前分析的目的，我们假设可见 sector 是可重整理论，并具体给出:

$$K\left(Z, Z^\dagger, \Phi, \Phi^\dagger\right) = \widehat{K}\left(Z, Z^\dagger\right) + \Phi^\dagger_{a^*} \Lambda^{a^*}{}_a \left(Z, Z^\dagger\right) \Phi^a + \left(\frac{1}{2} \Gamma_{ab}\left(Z, Z^\dagger\right) \Phi^a \Phi^b + \text{ h.c. }\right)$$

$$W(Z, \Phi) = \widehat{W}(Z) + \frac{1}{2} m_{ab}(Z) \Phi^a \Phi^b + \frac{1}{6} \lambda_{abc}(Z) \Phi^a \Phi^b \Phi^c \tag{19}$$

i.e., the Kähler potential (resp. superpotential) is a quadratic (resp. cubic) function of the observable fields, while the dependence of $\widehat{K}$ and $\widehat{W}$ or $\Lambda^{a^*}{}_a, \Gamma_{ab}, \lambda_{abc}, m_{ab}$ on the hidden fields remains arbitrary. Expanding the scalar potential is not intrinsically difficult, but the computation is lengthy because, as always in supergravity, there are numerous terms involved. We shall emphasize a few steps of this calculation (further technical details are given in [148]). Choosing an O'Raifeartaigh breaking mechanism, at least one of the F^i -fields in the hidden sector develops a vev:

即凯勒势 (对应超势) 是可观测场的二次 (对应三次) 函数，而 $\widehat{K}$ 和 $\widehat{W}$ 或 $\Lambda^{a^*}{}_a, \Gamma_{ab}, \lambda_{abc}, m_{ab}$ 对隐场的依赖仍保持任意性。展开标量势本身并不难，但计算过程冗长，因为和超引力中的所有情况一样，涉及大量项。我们仅强调该计算的几个关键步骤 (更多技术细节见文献 [148])。选择奥赖费尔陶破缺机制时，隐 sector 中至少有一个 F^i 场获得真空期望值:

$$F^i = \left\langle e^{\frac{1}{2}\frac{\widehat{K}}{m_p^2}} \widehat{K}^i{}_{I^*} \overline{\mathcal{D}}^{I^*} \widehat{W}^\star \right\rangle \neq 0, \tag{20}$$

(note that $\langle K \rangle \sim \langle \widehat{K} \rangle$ and $\langle W \rangle \sim \langle \widehat{W} \rangle$). The order parameter of supersymmetry breaking is then given by

(注意 $\langle K \rangle \sim \langle \widehat{K} \rangle$ 和 $\langle W \rangle \sim \langle \widehat{W} \rangle$)。超对称破缺的序参量则由下式给出

$$m^4_{\text{susy}} = \left\langle F^\dagger_{j^*} \right\rangle \left\langle \widehat{K}^{j^*}{}_i \right\rangle \left\langle F^i \right\rangle. \tag{21}$$

1. Inversion of the Kähler metric: Writing the Kähler metric and its inverse

1. 凯勒度量求逆: 写出凯勒度量及其逆

$$K = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} \partial_a \partial^{a^*} K & \partial_i \partial^{a^*} K \\ \partial_a \partial^{i^*} K & \partial_i \partial^{i^*} K \end{pmatrix}$$

$$K^{-1} = \begin{pmatrix} \left(A - BD^{-1}C\right)^{-1} & -\left(A - BD^{-1}C\right)^{-1} BD^{-1} \\ -D^{-1}C\left(A - BD^{-1}C\right)^{-1} & D^{-1}C\left(A - BD^{-1}C\right)^{-1} BD^{-1} + D^{-1} \end{pmatrix},$$

we have $A = A_0, B = m_p^{-1}B_1, C = m_p^{-1}C_1, D = m_p^{-2}D_2 + D_0$, with A_0, B_1, C_1, D_0, D_2 (not explicitly given here) of order m_p^0 , so that $D^{-1} = D_0^{-1}\left(1 + m_p^{-2}D_0D_2\right)^{-1}$ and $\left(A - BD^{-1}C\right)^{-1} = A_0^{-1}\left(1 - m_p^{-2}A_0B_1D^{-1}C_1\right)^{-1}$ are perturbatively inverted.

我们得到 $A = A_0, B = m_p^{-1}B_1, C = m_p^{-1}C_1, D = m_p^{-2}D_2 + D_0$ ，其中 A_0, B_1, C_1, D_0, D_2 (此处未明确给出) 是 m_p^0 阶的，因此 $D^{-1} = D_0^{-1}\left(1 + m_p^{-2}D_0D_2\right)^{-1}$ 和 $\left(A - BD^{-1}C\right)^{-1} = A_0^{-1}\left(1 - m_p^{-2}A_0B_1D^{-1}C_1\right)^{-1}$ 可通过微扰求逆。

2. One should compute all covariant derivatives $\mathcal{D}_I W = \partial_I W + m_p^{-2} W \partial_I K$ and all terms $\exp\left(m_p^{-2}K\right) \mathcal{D}_I W \bar{\mathcal{D}}^{J^*} W^\star K^I{}_{J^*}$ for $(I, J^*) = (i, j^*)$, (a, j^*) , (i, b^*) , and (a, b^*) corresponding to the hidden-hidden, observable-hidden, hidden-observable, and observable-observable sectors. We stress that these expressions all involve the inverse Kähler metric.

2. 需要分别计算对应隐-隐、可观测-隐、隐-可观测、可观测-可观测 sector 的所有协变导数 $\mathcal{D}_I W = \partial_I W + m_p^{-2} W \partial_I K$ 、所有项 $\exp\left(m_p^{-2}K\right) \mathcal{D}_I W \bar{\mathcal{D}}^{J^*} W^\star K^I{}_{J^*}$ (对应 $(I, J^*) = (i, j^*), (a, j^*), (i, b^*)$) 以及 (a, b^*) 。需要强调的是，所有这些表达式都包含逆凯勒度量。

3. Assume that the scalar fields z in the hidden sector are frozen to their vev $\langle z \rangle \sim m_p$, and introduce the gravitino mass (18). Note that, since in the expansion of $\mathcal{D}_I W K^i{}_{J^*} \mathcal{D}^{J^*} W^\star - 3m_p^{-2}|W|^2$, there are terms of order m_p^2 (see the cosmological constant below in V_{soft}), an additional contribution emerges from the expansion of the exponential factor (the term $1/m_p^2$ coming from the observable sector; see (19)).

3. 假设隐 sector 中的标量场 z 固定在其真空期望值 $\langle z \rangle \sim m_p$ ，并引入引力微子质量 (18)。注意，由于在 $\mathcal{D}_I W K^i{}_{J^*} \mathcal{D}^{J^*} W^\star - 3m_p^{-2}|W|^2$ 的展开中存在 m_p^2 阶项 (参见下文 V_{soft} 中的宇宙学常数)，指数因子的展开会产生额外贡献 (即可观测 sector 给出的项 $1/m_p^2$ ；参见 (19))。

Collecting all terms, the scalar potential takes the form [15, 148]

整理所有项后，标量势形式为 [15, 148]

$$V = m_p^2 \Lambda + \partial_a W_m \left\langle \left(\Lambda^{-1}\right)^a{}_{a^*} \right\rangle \partial^{a^*} W_m^\star + V_{\text{soft}} \tag{22}$$

to which the usual D -term (see (13)) should be added. The first term Λ (with a summation applying only to the fields of the hidden sector)

还需加上通常的 D 项 (参见 (13))。第一项 Λ (仅对隐 sector 的场求和)

$$\Lambda = \frac{1}{m_p^2} \left\langle F^\dagger_{j^*} \right\rangle \left\langle \hat{K}^{j^*} \right\rangle \left\langle F^i \right\rangle - 3\left| m_{\frac{3}{2}} \right|^2 = \frac{1}{m_p^2} m_{\text{susy}}^4 - 3\left| m_{\frac{3}{2}} \right|^2$$

is the cosmological constant. It is commonly accepted that, for phenomenologically acceptable theories, $\Lambda \approx 0$. Imposing this condition admittedly amounts to considerable fine-tuning. Assuming a vanishing cosmological constant leads to the relation

就是宇宙学常数。人们普遍认为，对于唯象学上可接受的理论，$\Lambda \approx 0$ 。不可否认，施加该条件意味着需要进行大量精细调节。假设宇宙学常数为零可得到如下关系

$$m_{\text{susy}}^4 = 3m_P^2 \left|m_{\frac{3}{2}}\right|^2. \tag{23}$$

The second term of (22) is the usual F -term of an unbroken supersymmetric theory with superpotential W_m given by

式 (22) 的第二项是未破缺超对称理论中常规的 F 项，该理论的超势 W_m 由下式给出

$$W_m = \frac{1}{6}\hat{\lambda}_{abc}\Phi^a\Phi^b\Phi^c + \frac{1}{2}\left(\widehat{m}_{ab} + m_{\frac{3}{2}}\langle\Gamma_{ab}\rangle - \left\langle F_{i^*}^{\dagger}\right\rangle\left\langle\partial^{i^*}\Gamma_{ab}\right\rangle\right)\Phi^a\Phi^b, \tag{24}$$

$$\hat{\lambda}_{abc} = e^{\frac{\langle\hat{K}\rangle}{2m_P^2}}\langle\lambda_{abc}\rangle \text{ and } \widehat{m}_{ab} = e^{\frac{\langle\hat{K}\rangle}{2m_P^2}}\langle m_{ab}\rangle. \tag{25}$$

Finally, the last term explicitly breaks supersymmetry:

最后一项显式破缺了超对称性:

$$V_{\text{soft}} = \varphi_{a^*}^{\dagger}\left(\left|m_{\frac{3}{2}}\right|^2 S^{a^*}{}_a + \Lambda\left\langle\Lambda^{a^*}{}_a\right\rangle\right)\varphi^a + \left(\frac{1}{6}A_{abc}\varphi^a\varphi^b\varphi^c + \frac{1}{2}B_{ab}\varphi^a\varphi^b + \text{ h.c. }\right),$$

where

其中

$$A_{abc} = e^{\frac{1}{2}\frac{\langle\hat{K}\rangle}{m_P^2}}\left\langle F^i\right\rangle\left[\frac{1}{m_P^2}\left\langle\partial_i\widehat{K}\right\rangle\langle\lambda_{abc}\rangle + \langle\partial_i\lambda_{abc}\rangle\right. \tag{26}$$

$$\left.-\left(\left\langle\left(\Lambda^{-1}\right)^d{}_{a^*}\partial_i\Lambda^{a^*}{}_a\lambda_{dbc}\right\rangle + (a\leftrightarrow b) + (a\leftrightarrow c)\right)\right]$$

for the trilinear terms,

对于三线性项,

$$B_{ab} = e^{\frac{1}{2}\frac{\langle\hat{K}\rangle}{m_P^2}}\left\langle F^i\right\rangle\left[\frac{1}{m_P^2}\left\langle\partial_i\widehat{K}\right\rangle\langle m_{ab}\rangle + \langle\partial_i m_{ab}\rangle - \left(\left\langle\left(\Lambda^{-1}\right)^c{}_{a^*}\partial_i\Lambda^{a^*}{}_b m_{ac}\right\rangle + (a\leftrightarrow b)\right)\right]$$

$$-m_{\frac{3}{2}}^{\dagger}e^{\frac{1}{2}\frac{\langle\hat{K}\rangle}{m_P^2}}\langle m_{ab}\rangle + \left(2\left|m_{\frac{3}{2}}\right|^2 + \Lambda\right)\langle\Gamma_{ab}\rangle - m_{\frac{3}{2}}^{\dagger}\left\langle F_{i^*}^{\star}\right\rangle\left\langle\partial^{i^*}\Gamma_{ab}\right\rangle \tag{27}$$

$$+m_{\frac{3}{2}}\left\langle F^i\right\rangle\left[\langle\partial_i\Gamma_{ab}\rangle - \left(\left\langle\left(\Lambda^{-1}\right)^c{}_{a^*}\partial_i\Lambda^{a^*}{}_b\Gamma_{ac}\right\rangle + (a\leftrightarrow b)\right)\right]$$

$$-\left\langle F^i F_{i^*}^{\star}\right\rangle\left[\left\langle\partial_i\partial^{i^*}\Gamma_{ab}\right\rangle - \left(\left\langle\left(\Lambda^{-1}\right)^c{}_{a^*}\partial_i\Lambda^{a^*}{}_b\partial^{i^*}\Gamma_{ac}\right\rangle + (a\leftrightarrow b)\right)\right].,$$

for the bilinear terms, and

对于双线性项，且

$$S^{a^*}{}_a = \langle \Lambda^{a^*}{}_a \rangle + \frac{1}{\left|m_{\frac{3}{2}}\right|^2} \left\langle F^\dagger_{i^*} \rangle \langle \partial^{i^*} \Lambda^{a^*}{}_b \Lambda^{-1} \right\rangle^b{}_{b^*} \partial_i \Lambda^{b^*}{}_a - \partial_i \partial^{i^*} \Lambda^{a^*}{}_a \Big\rangle \langle F^i \rangle \tag{28}$$

If (23) is satisfied, $\langle F^i \rangle \sim m_p m_{\frac{3}{2}}$, using (18),(20), and the scalings $\langle W \rangle \sim \langle \widehat{W} \rangle \sim \ M m_p^2, \langle K \rangle \sim \langle \widehat{K} \rangle \sim m_p^2$, and $\langle z \rangle \sim m_p$, one arrives at

若满足式 (23)，$\langle F^i \rangle \sim m_p m_{\frac{3}{2}}$ ，利用式 (18)、(20) 以及标度关系 $\langle W \rangle \sim \langle \widehat{W} \rangle \sim \ M m_p^2, \langle K \rangle \sim \langle \widehat{K} \rangle \sim m_p^2$ 和 $\langle z \rangle \sim m_p$ ，可得到

$$\langle F^i \rangle = m_p m^\dagger_{\frac{3}{2}} \left\langle \partial^{j^*} \left(\frac{1}{m_p} \widehat{K} + m_p \ln \frac{\widehat{W^\star}}{m_p^3} \right) \widehat{K}^i{}_{j^*} \right\rangle = m_p m^\dagger_{\frac{3}{2}} \hat{\rho}^i \tag{29}$$

with $\hat{\rho}^i \sim m_p^0$, and all soft supersymmetric scales are thus controlled by the gravitino mass, which is then required to be of the order of the electroweak scale (as an implementation of supersymmetry as a solution of the hierarchy problem).

其中 $\hat{\rho}^i \sim m_p^0$ ，所有软超对称破缺能标都由引力微子质量控制，因此要求引力微子质量的量级为电弱能标 (这是将超对称性作为等级问题解决方案的实现方式)。

As announced, the potential V_{soft} contains terms that break supersymmetry explicitly, but in a soft way. This means that the loop quantum corrections associated with these terms lead to at most logarithmic ultraviolet divergences. The last soft breaking term is the gaugino mass and is associated with a non-trivial gauge kinetic function $h_{ab} \neq \delta_{ab}$ (see, e.g.,[148]). We thus have four types of soft supersymmetric breaking terms:

如前所述，势 V_{soft} 包含了以软方式显式破缺超对称的项。这意味着与这些项相关的圈量子修正最多仅产生对数紫外发散。最后一个软破缺项是 gaugino 质量项，与非平庸规范动力学函数 $h_{ab} \neq \delta_{ab}$ 相关 (例如参见文献 [148])。因此我们共有四类软超对称破缺项:

- Squared masses for all the scalar fields (see the $S^{a^*}{}_b$ -terms)

- 所有标量场的质量平方项 (参见 $S^{a^*}{}_b$ 项)

- Masses for all gauginos (not derived here)

- 所有 gaugino 的质量项 (此处未推导)

- Trilinear couplings among the scalars (see the A_{abc} -terms)

- 标量之间的三线性耦合项 (参见 A_{abc} 项)

- Bilinear couplings among the scalars (see the B_{ab} -terms)

- 标量之间的双线性耦合项 (参见 B_{ab} 项)

The trilinear terms are directly related to the cubic part of the superpotential, while the bilinear terms have two origins: (1) one related to the quadratic part of the superpotential and (2) one related to the Γ -term in the Kähler potential. The contribution of the latter type is very interesting and allows to solve the famous μ -problem of the Minimal Supersymmetric Standard Model (see also the Γ_{ab} - contribution in (24)) [78]. (See section "Supergravity and Supersymmetry in Particle Physics.") In the presence of an additional linear term $\alpha_a \Phi^a$ in the superpotential (with Φ^a representing fields of the observable sector that are singlet under all gauge groups), the gravity-mediated supersymmetry breaking mechanism would have generated additional soft breaking terms of the form $C_a \varphi^a +$ h.c. The soft supersymmetric terms were classified in [76]. Terms of a similar form can be generated with the gauge and anomaly mediation mechanisms.

三线性项直接和超势的三次部分相关，而双线性项有两个起源:(1) 一个和超势的二次部分相关；(2) 一个和 Kähler 势中的 Γ 项相关。后者的贡献十分有意思，可以解决最小超对称标准模型中著名的 μ 问题 (也可参见式 (24) 中的 Γ_{ab} 贡献)[78]。(参见“粒子物理中的超引力与超对称性”一节。) 若超势中存在额外的线性项 $\alpha_a \Phi^a$ (其中 Φ^a 代表可观测部分中所有规范群下的单态场)，引力传递超对称破缺机制会产生形式为 $C_a \varphi^a +$ h.c. 的额外软破缺项。软超对称项已在文献 [76] 中完成分类。规范传递和反常传递机制也可以产生类似形式的项。

At the term of this construction, we recall that the effective Kähler potential for the observable fields Φ^a still reads $K_{\text{eff}} = \Phi^\dagger_{a^*} \langle \Lambda^{a^*} a \rangle \Phi^a$, so that the kinetic terms for the associated scalars and fermions are not canonically normalized. The last step for an application to particle physics thus amounts to a field re-definition $\Phi^a \to \hat{\Phi}^i$, enforcing the normalization $K_{\text{eff}} = \hat{\Phi}^\dagger_i \hat{\Phi}^i$ [148]. This operation further transforms the effective couplings applying to the $\hat{\Phi}^i$ basis, as compared to the expressions in Eqs. (22-28). In particular, the effective superpotential $W_m(\Phi^a) \to W_m^{\text{eff}}\left(\hat{\Phi}^i\right)$ then contributes to the scalar potential in the canonical form of Eq. (13), instead of that of Eq. (22) involving $\left\langle \left(\Lambda^{-1}\right)_a a^* \right\rangle$.

在该构造的末尾, 我们回顾: 可观测场 Φ^a 的有效凯勒势仍为 $K_{\text{eff}} = \Phi^\dagger_{a^*} \langle \Lambda^{a^*} a \rangle \Phi^a$, 因此关联标量和费米子的动力学项未按正则归一化。若要应用于粒子物理，最后一步因此需要做场重定义 $\Phi^a \to \hat{\Phi}^i$ ，强制满足归一化条件 $K_{\text{eff}} = \hat{\Phi}^\dagger_i \hat{\Phi}^i$ [148]。与式 (22-28) 中的表达式相比，该操作会进一步变换适用于 $\hat{\Phi}^i$ 基的有效耦合。特别地，有效超势 $W_m(\Phi^a) \to W_m^{\text{eff}}\left(\hat{\Phi}^i\right)$ 随后会以式 (13) 的正则形式对标量势产生贡献，而非包含 $\left\langle \left(\Lambda^{-1}\right)_a a^* \right\rangle$ 的式 (22) 的形式。

As a summary, let us briefly recapitulate the main steps entering the construction of a supergravity model exploitable in particle physics:

作为总结，我们简要概括构造可用于粒子物理的超引力模型的主要步骤:

1. Specify the observable gauge interactions and matter content, respectively, described by vector and chiral superfields.

1. 确定可观测规范相互作用和物质内容，二者分别由向量超场和手征超场描述。

2. Introduce the (gauge-invariant) basic functions of Eq. (19) fixing the interplay between observable and hidden sectors.

2. 引入式 (19) 的规范不变基本函数，确定可观测 sector 与隐 sector 之间的相互作用。

3. Induce spontaneous supersymmetry breaking in the hidden sector, and take the limit $m_p \to \infty$, which results in an effective field theory for the observable sector including soft supersymmetry breaking terms - see

Eqs. (22-28).

3. 在隐 sector 中诱导自发超对称破缺，并取极限 $m_p \to \infty$ ，最终得到可观测 sector 的有效场论，其中包含软超对称破缺项——见式 (22-28)。

4. Re-define the fields of the observable sector (via diagonalization and rescaling of $\langle\Lambda\rangle$) so that their kinetic terms are canonical.

4. 对可观测 sector 的场做重定义 (通过对 $\langle\Lambda\rangle$ 对角化和重标度)，使其动力学项满足正则形式。

We will directly exploit these results in section ”Supergravity and Supersymmetry in Particle Physics” when setting up a realistic supersymmetry-inspired model of particle physics, the Minimal Supersymmetric Standard Model.

在“粒子物理中的超引力与超对称”一节建立现实的、基于超对称的粒子物理模型——最小超对称标准模型时，我们会直接利用这些结果。

No-Scale Supergravity

无标度超引力

As mentioned in the previous section, gravity-mediated supersymmetry breaking suffers from a fine-tuning problem, related to the cosmological constant, which is artificially set to zero. In [28, 47, 112] , a class of models, known as no-scale supergravity, was derived, for which the vanishing of the cosmological constant emerges as a property of the underlying Kähler manifold where the scalar fields are living. Furthermore, the gravitino mass is generated dynamically. In fact, the scalar potential vanishes identically in such models (one speaks of flat directions in the potential). In particular, the gravitino mass is non-vanishing, although classically undetermined, and its values emerge through quantum corrections.

正如上一节所述，引力介导的超对称破缺存在精细调节问题，该问题与人为设为零的宇宙学常数相关。在 [28, 47, 112] 中，一类被称为无标度超引力的模型被推导出来，对于这类模型，宇宙学常数为零是标量场所在的底层凯勒流形的固有性质。此外，引力微子质量由动力学产生。事实上，这类模型中标量势恒为零 (即势中存在平坦方向)。特别地，引力微子质量非零，尽管经典层面无法确定，其值会通过量子修正得出。

Let us denote as $\Phi^I, I=(0,i)\,, i=1,\ldots,n$ the chiral superfields (Φ^0 being in the hidden sector and Φ^i in the observable sector). We first notice that it is possible to unify the Kähler potential and the superpotential into the so-called generalized Kähler potential

我们将手征超场记为 $\Phi^I, I=(0,i)\,, i=1,\ldots,n$ (Φ^0 属于隐藏扇区，Φ^i 属于可观测扇区)。我们首先注意到，可以将凯勒势和超势统一为所谓的广义凯勒势

$$\mathcal{G}\left(\Phi,\Phi^{\dagger}\right)=K\left(\Phi,\Phi^{\dagger}\right)+m_p^2\ln\frac{|W|^2}{m_p^6}.$$

In fact, this transformation corresponds to a superconformal transformation or a Kähler transformation (see, e.g., [148]). With this new function, the scalar potential takes the form

实际上，该变换对应共形变换或凯勒变换 (例如参见文献 [148])。利用这个新函数，标量势可写为如下形式

$$V = m_p^2 e^{\frac{1}{m_p^2}\mathcal{G}} \left(\mathcal{G}_i \mathcal{G}^i{}_{i*} \mathcal{G}^{i^*} - 3m_p^2\right),$$

with $\mathcal{G}_i = \partial_i \mathcal{G}, \mathcal{G}^{i^*} = \partial^{i^*}\mathcal{G}$, and $\mathcal{G}^{i^*}{}_j = \partial_i \partial^{i^*}\mathcal{G}$ the Kähler metric and $\mathcal{G}^i{}_{i*}$ its inverse.

其中 $\mathcal{G}_i = \partial_i \mathcal{G}, \mathcal{G}^{i^*} = \partial^{i^*}\mathcal{G}$ ， $\mathcal{G}^{i^*}{}_j = \partial_i \partial^{i^*}\mathcal{G}$ 是凯勒度量， $\mathcal{G}^i{}_{i*}$ 是其逆度量。

Choosing

若选取

$$\mathcal{G} = -3m_p^2 \ln\left[\frac{\phi^0 + \phi^\dagger_{0^*}}{m_p^2} - \frac{h\left(\phi^i, \phi^\dagger_{i^*}\right)}{m_p^2}\right] + m_p^2 F\left(\Phi^i\right) + m_p^2 F^*\left(\Phi^\dagger_{i^*}\right)$$

where h , which contributes to the kinetic part of the scalar φ^i and the fermions χ^i , is an unspecified function and F is a function related to interactions in the observable sector (in fact, one can show that $W = m_p^3 e^F$ with W the superpotential), a direct computation leads to $\mathcal{G}_i \mathcal{G}^i{}_{i*} \mathcal{G}^{i^*} = 3m_p^2$. Generalized Kähler potentials of the above type lead to no-scale models. Such models have been studied in the context of SU(5) grand unified theories in [47, 112] or in the context of the Standard Model [48].

其中 h 是一个未确定函数，它贡献标量 φ^i 和费米子 χ^i 的动能项，F 是与可观测扇区相互作用相关的函数 (实际上可以证明 $W = m_p^3 e^F$ ，其中 W 是超势)，直接计算可得 $\mathcal{G}_i \mathcal{G}^i{}_{i*} \mathcal{G}^{i^*} = 3m_p^2$ 。上述形式的广义凯勒势对应无标度模型，这类模型已在 [47, 112] 的 SU(5) 大统一理论框架，或标准模型框架下得到研究 [48]。

Supergravity and Supersymmetry in Particle Physics

粒子物理中的超引力与超对称

The formalism of the previous sections has provided us with a framework originally embedded in an $N = 1$ supergravity theory, but whose observable sector eventually reduces to an $N = 1$ supersymmetric model with explicit, albeit soft, supersym-metry breaking terms. The avowed purpose behind this construction rested with the double wish of, first, embedding the obviously non-supersymmetric spectrum of particle physics (”Standard Model”), at energies comparable to the electroweak scale, and, second, exploiting the technical protection of scalar masses against quantum corrections from the UV spectrum (Grand Unification, quantum gravity, etc.) in supersymmetric theories. The resulting hybrid indeed amounts to a non-supersymmetric model at energies below the scale of the supersymmetry breaking terms emerging in the observable sector, M_{soft} $\left(\approx m_{\frac{3}{2}}\right.$ in gravity mediation), but it retains the properties of a supersymmetric theory at energies above that scale. We shall now fulfil our program by explicitly introducing the Standard Model fields in this framework. The reader interested in a detailed discussion of supersymmetric extensions of the Standard Model will read with profit [43,44].

前面章节给出的形式体系为我们提供了一个最初嵌入 $N = 1$ 超引力理论的框架，但其可观测 sector 最终约化为带有显式 (尽管是软的) 超对称破缺项的 $N = 1$ 超对称模型。该构造的初衷基于两点期望: 首先，在电弱标度量级的能量下，嵌入粒子物理明显非超对称的能谱 (即“标准模型”)；其次，利用超对称理论中标量质量对来自紫外能谱 (大统一、量子引力等) 量子修正的技术性保护。由此得到的混合模型，确实在可观测 sector 中超对称破缺项的标度以下的能量区间成为一个非超对称模型 ($M_{\text{soft}} \left(\approx m_{\frac{3}{2}}\right.$ 在引力中介中)，但在该标度以上的能量区间仍保留超对称理论的性质。我们现在将推进我们的计划，在此框架中显式引入标准模型场。对标准模型超对称扩展的详细讨论感兴趣的读者可参考文献 [43,44]，定会有所收获。

The Minimal Supersymmetric Standard Model

最小超对称标准模型

Observing, first, that particle physics at energies comparable to the electroweak scale is well described by the Standard Model and, second, that M_{soft} should be relatively close to the electroweak scale in order to efficiently shield the Higgs squared mass (radiative corrections still involve the hierarchy between the electroweak and soft scales), it is meaningful to attempt an as-economical-as-possible embedding of the Standard Model in terms of new physics fields. The product of this operation is known as the Minimal Supersymmetric Standard Model (MSSM) [55, 56, 58, 90, 123] , and it will be the main model under discussion in this section. Following this principle of minimality, the gauge group remains unchanged as compared to the Standard Model, $G_{\text{SM}} = SU(3)_c \times SU(2)_L \times U(1)_Y$, but its implementation in a supersymmetric context calls for the introduction of the full set of vector superfields, gauge bosons, and associated gauginos, in the adjoint representation:

首先我们注意到，能量与电弱标度相当的粒子物理过程已被标准模型很好地描述；其次，M_{soft} 应当与电弱标度相当接近，才能有效屏蔽希格斯质量平方项 (辐射修正仍涉及电弱标度和软标度之间的层级差异)，因此我们有充分的理由尝试用尽可能精简的新物理场实现标准模型的嵌入。这一工作的产物就是最小超对称标准模型 (MSSM) [55, 56, 58, 90, 123] ，也是本节讨论的核心模型。遵循最小化原则，MSSM 的规范群与标准模型 $G_{\text{SM}} = SU(3)_c \times SU(2)_L \times U(1)_Y$ 相比没有发生改变，但要将它纳入超对称框架，就需要引入伴随表示下的全套矢量超场、规范玻色子以及对应的戈金诺:

$$SU(3)_c \rightarrow \widehat{G} = \left(G_\mu^a, \tilde{g}^a\right) \frac{T_a}{2} = \left(\widetilde{\mathfrak{L}}, \tilde{\mathbf{1}}, 0\right) = \text{ (gluons, gluinos)}$$

$$SU(2)_L \rightarrow \widehat{W} = \left(W_\mu^i, \widetilde{w}^i\right) \frac{\sigma_i}{2} = (\mathbb{1}, \mathbb{3}, 0) = (W \text{ - bosons, winos }) \tag{30}$$

$$U(1)_Y \rightarrow \widehat{B} = \left(B_\mu, \tilde{b}\right) = (1, 1, 0) = (B - \text{ boson, bino }) .$$

In the notation $(\mathbf{d_3}, \mathbf{d_2}, q_1)$, $\mathbf{d_3}, \mathbf{d_2}$, and q_1 correspond to the dimension of the $SU(3)_c$ representation, that of the $SU(2)_L$ representation, and the hypercharge under which the fields transform. The objects T_a and σ_i , respectively, denote the Gell-Mann and Pauli matrices. Superfields are written with a hat and supersymmetric partners with a tilde.

在该记法中，$(\mathbf{d}_3, \mathbf{d}_2, q_1)$, $\mathbf{d}_3, \mathbf{d}_2$ 、和 q_1 分别对应 $SU(3)_c$ 表示的维数、$SU(2)_L$ 表示的维数，以及场变换所属的超荷。对象 T_a 和 σ_i 分别指代盖尔曼矩阵和泡利矩阵。超场上方标注尖帽，超对称伙伴上方标注波浪号。

Similarly, the fermionic matter content of the Standard Model calls for the introduction of three generations ($f = 1, 2, 3$) of chiral superfields describing the quarks and leptons, as well as their supersymmetric counterparts, the squarks and sleptons. Writing only the left-handed fields (the right-handed ones are deduced by Hermitian conjugation; the superscript c is just a notation to distinguish $SU(2)_L$ singlets from the doublets):

类似地，标准模型的费米子物质内容要求引入三代 ($f = 1, 2, 3$) 手征超场，用以描述夸克、轻子以及它们的超对称对应粒子——超夸克和超轻子。此处仅写出左手场，右手场可通过厄米共轭导出；上标 c 只是用于区分 $SU(2)_L$ 单态与双态的记号:

$$\hat{Q}^f = \left(\begin{pmatrix}\tilde{u}^f \\ \tilde{d}^f\end{pmatrix}_L, \begin{pmatrix}u^f \\ d^f\end{pmatrix}_L\right) = \left(\overline{3}, \overline{2}, \ \frac{1}{6}\right), \hat{U}^{cf} = \left(\tilde{u}_R^{cf}, u_R^{cf}\right) = \left(\overline{3}, \overline{1}, -\frac{2}{3}\right),$$

$$\hat{D}^{cf} = \left(\tilde{d}_R^{cf}, d_R^{cf}\right) = \left(\overline{3}, \overline{1}, \ \frac{1}{3}\right),$$

(31)

$$\hat{L}^f = \left(\left(\tilde{\upsilon}^f\right)_L, \begin{pmatrix}\upsilon^f \\ e^f\end{pmatrix}_L\right) = \left(1, 2, -\frac{1}{2}\right), \hat{E}^{cf} = \left(\tilde{e}_R^{cf}, e_R^{cf}\right) = (1, 1, 1).$$

Finally, the Higgs field requires another chiral supermultiplet $\hat{H} = \left(1, 2, -\frac{1}{2}\right)$, which it shares with the higgsino. Yet, the above field content would lead to non-vanishing chiral anomalies, hence to a non-renormalizable model, due to the addition of the electroweakly charged higgsino to the fermion sector of the Standard Model. In addition, the holomorphicity of the superpotential forbids the use of conjugate fields when writing the Yukawa terms, so that top and bottom masses cannot be generated with a single Higgs superfield. This prompts for the introduction of two Higgs supermultiplets with opposite hypercharges instead:

最后，希格斯场还需要另一个手征超多重态 $\hat{H} = \left(1, 2, -\frac{1}{2}\right)$，该多重态由希格斯场与希格斯微子共有。但上述场配置会产生非零手征反常，使得模型无法重整，原因是标准模型的费米子 sector 中新增了带电弱电荷的希格斯微子。此外，超势的全纯性要求我们在书写汤川项时不能使用共轭场，因此单个希格斯超场无法同时生成顶夸克质量和底夸克质量。这就要求我们引入两个超荷相反的希格斯超多重态，如下所示:

$$\hat{H}_D = \left(\begin{pmatrix}H_D^0 \\ H_D^-\end{pmatrix}, \begin{pmatrix}\tilde{h}_D^0 \\ \tilde{h}_D^-\end{pmatrix}\right) = \left(1, 2, -\frac{1}{2}\right), \hat{H}_U = \left(\begin{pmatrix}H_U^+ \\ H_U^0\end{pmatrix}, \begin{pmatrix}\tilde{h}_U^+ \\ \tilde{h}_U^0\end{pmatrix}\right) = \left(1, 2, \frac{1}{2}\right).$$

(32)

For the Higgs fields, we have further indicated their $U(1)_{\text{em}}$ -charges.

对于希格斯场，我们还进一步标出了它们的 $U(1)_{\text{em}}$ 荷。

The chiral superfields $\hat{Q}^f, \hat{U}^{cf}, \hat{D}^{cf}, \hat{L}^f, \hat{E}^{cf}, \hat{H}_U$, and $\hat{H}_D$ represent, for the MSSM, the observable fields denoted as $\hat{\Phi}^i$ at the end of the construction of section "Gravity-Induced Supersymmetry Breaking" - i.e., after enforcing a canonical normalization of the kinetic terms. Following the results of Eqs. (22-28), the interactions in the observable sector of the MSSM can be described by:

在手征超场 $\hat{Q}^f, \hat{U}^{cf}, \hat{D}^{cf}, \hat{L}^f, \hat{E}^{cf}, \hat{H}_U$ 和 $\hat{H}_D$ 表示 MSSM 中，在“引力诱导超对称破缺”一节构造完成后 (即在对动力学项规定标准归一化之后) 得到的可观测场，记为 $\hat{\Phi}^i$ 。根据式 (22-28) 的结果，MSSM 可观测部分的相互作用可以描述为:

- An effective (by assumption renormalizable) superpotential (we stress that the matrix $\left\langle \left(\Lambda^{-1}\right)^a{}_{a^*} \right\rangle$ appearing in Eq. (22) becomes trivial after the field redefinition restoring canonical kinetic terms), constrained by the symmetries

- 一个有效的 (假设可重整的) 超势 (我们要强调: 式 (22) 中出现的矩阵 $\left\langle \left(\Lambda^{-1}\right)^a{}_{a^*} \right\rangle$ 在重定义场恢复正则动能项后会变得平凡)，受对称性约束

$$W_{\text{MSSM}} = \mu \hat{H}_U \cdot \hat{H}_D + Y_u^{ff'} \hat{Q}^f \cdot \hat{H} \hat{U}^{cf'} - Y_d^{ff'} \hat{Q}^f \cdot \hat{H}_D \hat{D}^{cf'} - Y_e^{ff'} \hat{L}^f \cdot \hat{H}_D \hat{E}^{cf'}$$

$$+\mu_i \hat{H}_U \cdot \hat{L}^i + \lambda_{ijk} \hat{L}^i \cdot \hat{L}^j \hat{E}^{ck} + \lambda'_{ijk} \hat{L}^i \cdot \hat{Q}^j \hat{D}^{ck} + \lambda''_{ijk} \hat{U}^{ci} \hat{D}^{cj} \hat{D}^{ck}$$

(33)

where $\Phi^a \cdot \Phi^b \equiv (\Phi^a)_1 (\Phi^b)_2 - (\Phi^a)_2 (\Phi^b)_1$ stands for the $SU(2)_L$ -invariant product (with the indices 1 and 2 referring to the weak isospin)

其中 $\Phi^a \cdot \Phi^b \equiv (\Phi^a)_1 (\Phi^b)_2 - (\Phi^a)_2 (\Phi^b)_1$ 代表 $SU(2)_L$ 不变乘积 (指标 1 和 2 对应弱同位旋)

- A set of soft breaking terms of similar form, with bilinear and trilinear scalar couplings B s and A s as in Eqs. (26 and 27)

- 一组形式类似的软破缺项，包含双线性和三线性标量耦合 B s 和 A s ，形式如式 (26) 和 (27)

- Squared mass terms $m^2{}_a^{a^*} \hat{\phi}^\dagger_{a^*} \hat{\phi}^a$ - determined by Eq. (28) - where off-diagonal elements only occur for scalar fields with identical gauge quantum numbers

- 平方质量项 $m^2{}_a^{a^*} \hat{\phi}^\dagger_{a^*} \hat{\phi}^a$ ——由式 (28) 确定——仅规范量子数相同的标量场才存在非对角元

- Gaugino mass terms $\frac{1}{2} M_1 \tilde{b} \cdot \tilde{b} + \frac{1}{2} M_2 \widetilde{w}_i \cdot \widetilde{w}_i + \frac{1}{2} M_3 \tilde{g}_a \cdot \tilde{g}_a +$ h.c.

- 戈迹子质量项 $\frac{1}{2} M_1 \tilde{b} \cdot \tilde{b} + \frac{1}{2} M_2 \widetilde{w}_i \cdot \widetilde{w}_i + \frac{1}{2} M_3 \tilde{g}_a \cdot \tilde{g}_a +$ h.c.

Also, the D -term contribution to the scalar potential should be restored in Eq. (22). Then, given the arbitrariness of the functions $\hat{K}, \Lambda$, and Γ in the hidden sector, as well as the possibility to call to various breaking mechanisms beyond gravity mediation, it is evident from Eqs. (26-28) that almost any choice of soft breaking parameters can be a posteriori justified by a judicious Ansatz. The simplistic case of trivial functions leads to universality conditions (the flavor index is omitted):

此外，式 (22) 中应补上 D 项对标量势的贡献。随后，由于隐藏区中函数 $\hat{K}, \Lambda$ 和 Γ 具有任意性，且除引力介导外还存在多种破缺机制，从式 (26-28) 中可以明显看出: 几乎任意软破缺参数的选择都可以通过一个恰当的假设事后得到辩护。平凡函数的简单情况会给出普适性条件 (味指标已省略):

$$V_{\text{soft}}^{\text{cMSSM}} = m_0^2\left(|H_U|^2 + |H_D|^2 + |Q|^2 + |U^c|^2 + |D^c|^2 + |L|^2 + |E^c|^2\right)$$

$$+\frac{1}{2}M_{\frac{1}{2}}\left(\tilde{b}\cdot\tilde{b} + \widetilde{w}_i\cdot\widetilde{w}_i + \tilde{g}_a\cdot\tilde{g}_a + \text{ h.c. }\right)$$

$$+\left[b\mu H_U\cdot H_D + a\left(Y_u Q\cdot H_U U - Y_d Q\cdot H_D D^c - Y_e L\cdot H_D E^c\right) + \text{ h.c. }\right].$$

(34)

This choice, (when the second line of (33) is not considered) more predictive than realistic, is only marginally compatible with experimental results today [12,91].

这种选择 (不考虑式 (33) 第二行时) 更偏向预测性而非现实性，仅勉强符合当前的实验结果 [12,91]。

A discrete symmetry, R -parity [53], with charge assignment 1 to all Standard Model fields and -1 to their supersymmetric partners, is often introduced to eliminate the terms of the second line of Eq. (33), as well as the associated soft breaking couplings. These terms explicitly violate lepton or baryon number and lead to a distinctive phenomenology [10]. We discard them till further notice and work with the reduced superpotential $W_{\text{MSSM}}^{\text{RpC}}$ defined by the first line of Eq. (33). An alternative charge assignment, actually equivalent, is that of a matter parity, with all quark and lepton superfields carrying a charge -1, while the Higgs superfields transform trivially.

人们通常引入一个离散对称性，即 R 宇称 [53]，规定所有标准模型场的宇称荷为 1，其超对称伙伴的宇称荷为-1，用来消去式 (33) 第二行的项以及对应的软破缺耦合。这些项明确破坏轻子数或重子数，会带来独特的唯象学 [10]。在另行通知前我们舍弃这些项，使用由式 (33) 第一行定义的约化超势 $W_{\text{MSSM}}^{\text{RpC}}$ 。另一种实际上等价的荷分配是物质宇称: 所有夸克和轻子超场带电荷-1，希格斯超场为平凡变换。

Let us pause at this point and look at the spectrum in the absence of electroweak symmetry breaking. Protected by G_{SM} , the quarks, leptons, and gauge bosons remain massless, as expected. Squarks and sleptons receive squared masses $\sim M_{\text{soft}}^2$ from the quadratic soft terms: the absence of observed resonances at the Large Hadron Collider tends to push M_{soft} above the TeV scale. The gauginos take masses of order M_{soft} from the soft breaking parameters $M_{1,2,3}$, while higgsinos receive a mass μ from the superpotential. Several squared mass scales intervene in the Higgs sector, μ^2 from the superpotential and $B \sim \mu M_{\text{soft}}$ and $m_{H_{U,D}}^2 \sim M_{\text{soft}}^2$ from the soft breaking scalar potential: their interplay must be examined more closely to deduce the conditions of emergence for electroweak symmetry breaking.

我们在此稍作停顿，考察未发生电弱对称性破缺时的能谱。在 G_{SM} 的保护下，夸克、轻子和规范玻色子如预期般保持无质量状态。squarks 和 sleptons 从软二次项获得质量平方 $\sim M_{soft}^2$: 大型强子对撞机尚未观测到相关共振，这一结果倾向于将 M_{soft} 推至 TeV 能标以上。缪微子 (gauginos) 从软破缺参数 $M_{1,2,3}$ 获得量级为 M_{soft} 的质量，而希格斯微子 (higgsinos) 从超势获得质量 μ 。希格斯 sector 涉及多个质量平方标度: μ^2 来自超势，$B \sim \mu M_{soft}$ 和 $m_{H_{U,D}}^2 \sim M_{soft}^2$ 来自软破缺标量势: 为了推导电弱对称性破缺发生的条件，我们需要更细致地分析它们的相互作用。

The μ -Problem

μ 问题

It was realized early on [109] that the presence of a supersymmetry-conserving mass term μ in the superpotential of Eq. (33) was phenomenologically problematic. This parameter being a priori unrelated to supersymmetry breaking, a natural choice for μ would involve some high-energy scale, Planck or Grand Unification, much above M_{soft} . However, in such a case, the Higgs potential is dominated by the squared mass term $|\mu|^2 \left(|H_D|^2 + |H_U|^2\right)$, which no soft contribution can balance, and the electroweak symmetry cannot be broken. Failing to have μ large, we may set it to 0, such a choice being protected by the emergence of a $U(1)$ symmetry. Nevertheless, given that the higgsinos take their mass from μ and that none was observed at the Large Electron-Positron Collider, $\mu \geq 100\text{GeV}$, which invalidates this alternative. The non-renormalization theorems [87] also forbid to generate the μ parameter radiatively. In the aftermath, the unnatural choice $\mu \approx M_{soft}$ must be retained: this is the μ -problem.

人们很早就意识到 [109]，式 (33) 超势中存在超对称守恒质量项 μ 在唯象上存在问题。该参数本来就和超对称破缺无关，因此 μ 的自然取值会对应普朗克能标或大统一能标这类远高于 M_{soft} 的高能标。但在这种情况下，希格斯势会被平方质量项 $|\mu|^2 \left(|H_D|^2 + |H_U|^2\right)$ 主导，没有软项可以抵消这一项，电弱对称性就无法破缺。如果不把 μ 取大，我们可以将其设为 0，该选择会因 $U(1)$ 对称性的出现得到保护。然而，由于希格斯诺的质量来自 μ ，且大型正负电子对撞机并未观测到希格斯诺，因此 $\mu \geq 100\text{GeV}$ ，这一备选方案不成立。非重整化定理 [87] 也禁止通过辐射修正产生 μ 参数。最终只能保留不自然的取值 $\mu \approx M_{soft}$: 这就是 μ 问题。

A first solution consists in relating the emergence of μ to the supersymmetry breaking mechanism: this is the proposal by Giudice and Masiero [78]. In this approach, the μ parameter is absent in the original superpotential of the supergravity model (all parameters are then dimensionless in the Kähler potential and the superpotential (19)), while a term $\Gamma\left(Z, Z^\dagger\right) \widehat{H}_U \cdot \widehat{H}_D$ appears in the Kähler potential - thus explicitly breaking the $U(1)$ symmetry and avoiding the appearance of an associated Goldstone boson when Higgs fields take a vev. Then the "observable" μ parameter appears in the limit $m_p \to \infty$ through the Γ contributions to the bilinear terms in Eq. (24) and is naturally of the order of the gravitino mass. A variant [20] consists in writing a term $\frac{1}{m_p^2} \lambda(Z) \widehat{W}(Z) H_U \cdot H_D$ in the superpotential: it can be related to a choice $\Gamma\left(Z, Z^\dagger\right) = \lambda(Z) + \lambda^\star\left(Z^\dagger\right)$ by a Kähler transformation.

第一种解决方案将 μ 的产生和超对称破缺机制关联起来: 这是 Giudice 和 Masiero 提出的方案 [78]。在该方法中，超引力模型的原始超势不存在 μ 参数 (此时凯勒势和超势 (19) 中所有参数都是无量纲的)，而凯勒势中会出现项 $\Gamma\left(Z, Z^{\dagger}\right) \widehat{H}_U \cdot \widehat{H}_D$，由此明确破缺 $U(1)$ 对称性，避免希格斯场获得真空期望值时出现相伴的戈德斯通玻色子。之后，可观测的 μ 参数会在 $m_p \rightarrow \infty$ 极限下，通过式 (24) 双线性项的 Γ 贡献产生，其自然大小为引力微子质量量级。一个变体方案 [20] 是在超势中写下项 $\frac{1}{m_p^2} \lambda(Z) \widehat{W}(Z) H_U \cdot H_D$: 它可以通过凯勒变换对应到选择 $\Gamma\left(Z, Z^{\dagger}\right)=\lambda(Z)+\lambda^{\star}\left(Z^{\dagger}\right)$。

Another solution consists in generating the μ parameter directly in the observable sector from the vev of an additional singlet superfield $\hat{S}=(1,1,0)$ [54]. The corresponding model is known as the Next-to-Minimal Supersymmetric Standard Model (NMSSM) [50,51]. The simplest choice to forbid the μ term in the superpotential amounts to a $\mathbb{Z}_3$ symmetry, allowing only for cubic terms:

另一种解决方案是通过额外的单态超场 $\hat{S}=(1,1,0)$ 的真空期望值，直接在可观测 sector 中产生 μ 参数 [54]。对应的模型称为次最小超对称标准模型 (NMSSM)[50,51]。禁止超势中出现 μ 项的最简单选择是引入一个 $\mathbb{Z}_3$ 对称性，仅允许三次项:

$$W_{\mathrm{NMSSM}}^{\mathbb{Z}_3}=\lambda \hat{S} \widehat{H}_U \cdot \widehat{H}_D+\frac{\kappa}{3} \hat{S}^3+\text { Yukawas }, \tag{35}$$

where "Yukawas" stands for the first line in (33) without the μ -term. Then the minimization of the scalar potential generally allows S to develop a vev $\langle S\rangle$, which results in an effective μ parameter $\mu_{\text{eff}}=\lambda\langle S\rangle$. We stress that the only scale entering the scalar potential is the supersymmetry breaking one, thus naturally relating the electroweak symmetry breaking scale to the latter. In addition, there are numerous phenomenological applications of the new singlet fields, on which we will opportunistically comment in due time. On the other hand, the $\mathbb{Z}_3$ symmetry proves problematic from the perspective of cosmology, where it causes a domain wall problem [149]. Alternative choices using R -symmetries have been studied in [113].

其中“汤川耦合项”指式 (33) 第一行中去掉 μ 项后的部分。此时对标量势做最小化一般会允许 S 获得真空期望值 $\langle S\rangle$，从而得到有效 μ 参数 $\mu_{\text{eff}}=\lambda\langle S\rangle$。我们需要强调，进入标量势的唯一能标是超对称性破缺能标，因此电弱对称性破缺能标自然与该能标联系起来。此外，新增的单态场还有大量唯象学应用，我们会适时在合适的时机对此展开讨论。另一方面，$\mathbb{Z}_3$ 对称性从宇宙学的角度来看存在问题，它会引发畴壁问题 [149]。采用 R 对称性的替代方案已在文献 [113] 中被研究。

Radiative Corrections and Renormalization Group Evolution

辐射修正与重整化群演化

Meaningful predictions in the MSSM (or any supersymmetric extension of the Standard Model) imply processing this framework as a quantum field theory, i.e., being able to calculate quantum corrections. To this end, it is necessary to regularize ultraviolet divergences appearing in loop diagrams and re-define bare parameters so that such divergences cancel out at the level of observable quantities (renormalization). The most popular approach to regularization consists in performing calculations in $D=4-2\varepsilon$ spacetime dimensions instead of 4 . However, this "naive" dimensional regularization explicitly violates supersymmetry in that vector fields become $4-2\varepsilon$ -dimensional, while their fermionic counterparts remain 4- dimensional, hence introducing a mismatch between bosonic and fermionic degrees of freedom. This issue is addressed in dimensional reduction [137, 144], where vector fields formally retain 4 dimensions through the introduction of ”epsilon scalars,” living in the 2ε dimensions. It is then possible to renormalize the model, e.g., through modified minimal subtraction of the ultraviolet divergences, leading to the $\overline{\text{DR}}$ scheme. Alternatively, the $\overline{\text{DR}}'$ scheme [101] allows to decouple the epsilon scalars. We stress that it is not absolutely imperative to work in a framework respecting supersymmetry. One then forfeits the relations between parameters (e.g., in the scalar and fermion interactions) that are guaranteed by supersymmetry. Thus, MS, for example, remains a legitimate choice. Nevertheless, to ensure that this non-supersymmetric treatment describes a (softly broken) supersymmetric model, it is ultimately necessary to define its renormalized parameters through their connection to those of a supersymmetry-conserving scheme.

> 在 MSSM(或任何超对称标准模型扩展) 中获得有意义的预言，需要将该框架作为量子场论处理，即能够计算量子修正。为此，必须对圈图中出现的紫外发散进行正规化，重新定义裸参数，使这类发散在可观测量层面抵消 (即重整化)。最常用的正规化方法是在 $D = 4 - 2\varepsilon$ 维时空而非四维时空进行计算。但这种“朴素”维正规化明显破坏超对称性: 矢量场变为 $4 - 2\varepsilon$ 维，而其对应费米子仍保持四维，这就导致玻色子与费米子的自由度不匹配。维约化方案 [137, 144] 解决了这一问题: 在维约化中，通过引入存活于 2ε 额外维的“ε 标量”，矢量场形式上保留四维。随后我们就可以对模型进行重整化，例如通过对紫外发散做修正最小减除，得到 $\overline{\text{DR}}$ 方案。另一种方案 $\overline{\text{DR}}'$ [101] 可以对 ε 标量退耦。需要强调的是，在计算中并非必须采用尊重超对称性的框架。不遵守超对称性的做法只是会失去超对称性保证的参数间关系 (例如标量与费米子相互作用的关系)，因此诸如 MS 这类方案仍然是合理选择。不过，若要保证这种非超对称处理描述的是 (软破缺) 超对称模型，最终仍需要通过超对称守恒方案的参数来定义自身的重整化参数。

In a context involving vastly different scales, such as electroweak physics on the one side and gravity-mediated supersymmetry breaking on the other, it is preferable to resum the ultraviolet logarithms developing between the two scales through the use of the renormalization group evolution, rather than work with parameters defined at a widely different energy from that of the physical process under study. This coming-and-going between scales is therefore needed, both for testing the predictions of a given high-energy model, e.g., of supersymmetry breaking, on particle physics and in view of inferring the structure of the high-energy theory from the low-energy phenomenology. The beta functions of gauge couplings and superpotential parameters are known up to four and three loops, respectively [102-105]. On the other hand, the renormalization group equations for the soft breaking parameters are generically known at two-loop order [114, 159], although methods have been proposed and implemented in special cases to include higher orders. As could be anticipated, the running of gauge and superpotential parameters involves only supersymmetry-conserving parameters in their beta functions, while the equations for the soft parameters include both soft breaking and supersymmetry-conserving parameters. The impact of the running for well-separated scales is considerable in general, and a set of degenerate soft mass parameters at m_P , as in Eq. (34), or at a Grand Unification scale - at which several fields are connected by the extended gauge symmetry - would produce a hierarchical spectrum at the TeV scale.

当问题涉及差异极大的能标 (例如一端是电弱物理，另一端是引力传递的超对称破缺)，更优的做法是利用重整化群演化对两个能标之间演化出的紫外对数做重求和，而非采用与所研究物理过程能量相差极远的参数来计算。这种在不同能标间转换的操作是必要的: 既用于检验特定高能模型 (例如超对称破缺模型) 的粒子物理预言，也用于从低能唯象学反推高能理论的结构。规范耦合和超势参数的 β 函数分别已知到四圈和三圈精度 [102-105]。而软破缺参数的重整化群方程一般已知到两圈阶 [114, 159]，不过已有方法在特殊情形下提出并实现了更高阶的包含。正如预期，规范参数和超势参数 β 函数中仅包含超对称守恒参数，而软参数的方程同时包含软破缺参数和超对称守恒参数。一般而言，能标分隔较大时，跑动的影响相当显著: 如式 (34) 所示，若 m_p 处软质量参数简并，或是在大统一能标 (扩展规范对称性将多个场联系在此能标) 处软质量参数简并，最终在 TeV 能标会得到层次化能谱。

As an interesting consequence of the modified matter content of the MSSM, with new physics fields in the TeV range, the gauge couplings of G_{SM} show an approximate convergence when running them up from the electroweak scale toward a unification scale of $M_{\text{GUT}} \approx 10^{16}$GeV [4]. This feature would allow for a Grand Unification of G_{SM} in a single step.

MSSM 修改了物质内容，在 TeV 能区引入了新物理场，一个有趣的结果是: 从电弱标度向上跑动到 $M_{\text{GUT}} \approx 10^{16}$GeV 量级的统一能标时，$G_{\text{SM}}$ 的规范耦合近似收敛 [4]。这一特性允许 G_{SM} 一步完成大统一。

Electroweak Symmetry Breaking and the MSSM Higgs Sector

电弱对称性破缺与 MSSM 希格斯场

The tree-level scalar potential for the Higgs fields in the MSSM reads

MSSM 中希格斯场的树层级标量势表达式如下

$$\mathcal{V}_H = \left(m_{H_D}^2 + |\mu|^2\right)|H_D|^2 + \left(m_{H_U}^2 + |\mu|^2\right)|H_U|^2 + \left[m_{12}^2 H_U \cdot H_D + \text{ h.c. }\right]$$

$$+\frac{g_1^2+g_2^2}{8}\left[|H_D|^4 + |H_U|^4\right] + \frac{g_2^2-g_1^2}{4}|H_D|^2|H_U|^2 - \frac{g_2^2}{2}|H_U \cdot H_D|^2$$

(36)

where g_1 and g_2 are the electroweak gauge couplings, from which one can infer the D -term origin of the quartic operators. The mass terms $m_{H_{D,U}}^2 \sim M_{\text{soft}}^2$ and $m_{12}^2 \sim M_{\text{soft}}\,\mu$ (substituting the " B " notation) denote the quadratic and bilinear soft breaking terms, respectively. Any complex phase in m_{12}^2 can be absorbed through a re-definition of the (super)fields, so that the tree-level Higgs potential of the MSSM is automatically CP-conserving - this is no longer necessarily the case in extended models such as the NMSSM.

其中 g_1 和 g_2 是电弱规范耦合，由此可以推导出四次算符的 D 项起源。质量项 $m_{H_{D,U}}^2 \sim M_{\text{soft}}^2$ 和 $m_{12}^2 \sim M_{\text{soft}}\,\mu$ (替代“B ”记号) 分别表示二次和双线性软破缺项。m_{12}^2 中的任何复相位都可以通过重新定义 (超) 场吸收，因此 MSSM 的树级希格斯势自动满足 CP 守恒——这在 NMSSM 等推广模型中不再必然成立。

The minimization of Eq. (36) (see the details in, e.g., [40, 73]) leads to the solution $\left(\langle H_D^0\rangle, \langle H_U^0\rangle\right) = v\,(\cos\beta, \sin\beta)$ respecting the electromagnetic $U(1)$ symmetry under some conditions among the parameters. The parameter v should be identified to the electroweak symmetry breaking vev $\left(2\sqrt{2}G_F\right)^{-1/2}$, where G_F is the Fermi constant measured in muon decays. Then, after examining the quadratic terms in the vicinity of this minimum, a triplet of Goldstone bosons G^0 and $G^\pm$ decouples, leaving a CP-odd mass eigenstate A^0 with squared mass $M_A^2 \equiv \frac{2m_{12}^2}{\sin 2\beta}$ and a charged one $H^\pm$, with $M_{H^\pm}^2 = M_A^2 + M_W^2$ and $M_W^2 \equiv \frac{g_2^2}{2}v^2$. Two states h^0 and H^0 remain in the CP-even sector and mix according to the mass matrix ($M_{W,Z}$ are the masses of the electroweak gauge bosons):

对式 (36) 做极小化 (例如参见文献 [40, 73] 了解细节)，得到满足参数间条件下保持电磁 $U(1)$ 对称性的解 $\left(\langle H_D^0\rangle, \langle H_U^0\rangle\right) = v\,(\cos\beta, \sin\beta)$。参数 v 对应电弱对称性破缺的真空期望值 $\left(2\sqrt{2}G_F\right)^{-1/2}$，其中 G_F 是缪子衰变测得的费米常数。随后，分析该极小值邻域内的二次项后，戈德斯通玻色子三重态 G^0 和 $G^\pm$ 退耦，留下一个 CP 奇质量本征态 A^0，其平方质量为 $M_A^2 \equiv \frac{2m_{12}^2}{\sin 2\beta}$，以及一个带电希格斯 $H^\pm$，对应 $M_{H^\pm}^2 = M_A^2 + M_W^2$ 和 $M_W^2 \equiv \frac{g_2^2}{2}v^2$。CP 偶区剩下两个态 h^0 和 H^0，按照质量矩阵 ($M_{W,Z}$ 混合 (其中 ($M_{W,Z}$ 是电弱规范玻色子的质量):

$$\mathcal{M}_{\mathrm{CPE}}^2 \equiv \begin{bmatrix} M_A^2\sin^2\beta + M_Z^2\cos^2\beta - \left(M_A^2 + M_Z^2\right)\sin\beta\cos\beta \\ -\left(M_A^2 + M_Z^2\right)\sin\beta\cos\beta M_A^2\cos^2\beta + M_Z^2\sin^2\beta \end{bmatrix}; M_Z^2 \equiv \frac{g_1^2 + g_2^2}{2}v^2.$$

(37)

In this equation, the minimization of the Higgs potential has been explicitly applied. The reader may consult [73] for an explicit computation of the various mass matrix terms.

本式已经显式应用了希格斯势的极小化条件。读者可查阅文献 [73] 获取各质量矩阵项的具体计算过程。

An effective Standard Model is obtained in the limit $M_A \gg M_Z$, where h^0 exactly identifies with the electroweak partner of the Goldstone bosons, while $\left(H^0, A^0, H^\pm\right)$ form a heavy (degenerate) doublet. In view of the measured properties of the Higgs particle at 125.25 GeV, as well as the phenomenological constraints on non-standard Higgs doublets, this decoupling scenario appears as the most realistic one. Nevertheless, light non-standard Higgs states are still very compatible with collider data as long as they are dominantly singlet, as happens in, e.g., the NMSSM, due to their suppressed coupling to standard matter.

当满足 $M_A \gg M_Z$ 极限时，可以得到有效标准模型，此时 h^0 与戈德斯通玻色子的电弱伙伴完全对应，而 $\left(H^0, A^0, H^\pm\right)$ 构成一个简并重双重态。结合 125.25 GeV 希格斯粒子的已测性质，以及非标准希格斯双重态的唯象约束，这种退耦情景是目前最符合实际的情况。不过，只要非标准轻希格斯态主要是单态，就仍然符合对撞机数据，例如 NMSSM 中的情况，原因是这类态与标准物质的耦合被压低。

The lightest eigenvalue of Eq. (37) satisfies $m_{h^0}^2 \leq \min\left(M_Z^2, M_A^2\right)\cos^2 2\beta$. Given $M_Z \approx 91\mathrm{GeV}$, m_{h^0} seems much below the observed 125.25 GeV. However, this represents no fundamental incompatibility, because the connection between m_{h^0} and M_Z is a tree-level relation enforced by supersymmetry, but, the latter being broken at the electroweak scale, it is not preserved by radiative corrections. The estimated upper bound on the physical mass M_{h^0} before the Higgs discovery amounted to $M_{h^0} \leq 135\mathrm{GeV}$ in the MSSM (strongly dependent on the mass of the top quark m_t) [40]. Nevertheless, $\tan\beta \gg 1$ is favored in order to maximize the tree-level contribution. Further tree-level effects are possible in extensions of the MSSM, such as a contribution of the F -term λ (see Eq. (35)) in the NMSSM or a mass uplift via mixing of h^0 with a lighter CP-even singlet. Still, radiative corrections to the mass of the Standard Model-like Higgs reach a considerable relative size, so that a good control of the higher orders is needed to reduce the uncertainties. The corresponding calculations are reviewed in [139]. The leading corrections are controlled by the Yukawa coupling of the top:

式 (37) 的最小本征值满足 $m_{h^0}^2 \leq \min\left(M_Z^2, M_A^2\right)\cos^2 2\beta$ 。给定 $M_Z \approx 91\text{GeV}, m_{h^0}$ 后，结果远低于观测值 125.25 GeV。但这并不存在根本的不相容性，因为 m_{h^0} 与 M_Z 之间的关联是超对称要求的树水平关系，而超对称在电弱标度处已经破缺，因此该关系不会被辐射修正保留。在希格斯玻色子发现之前，MSSM 中物理质量 M_{h^0} 的估计上限为 $M_{h^0} \leq 135\text{GeV}$ (该结果强烈依赖于顶夸克质量 m_t)[40]。尽管如此，为了最大化树水平贡献，仍倾向于大 $\tan\beta \gg 1$ 。在 MSSM 的扩展中可以存在额外的树水平效应，例如 NMSSM 中 F 项 λ 的贡献 (见式 (35))，或是通过 h^0 与更轻 CP-even 单态的混合提升质量。即便如此，对类标准模型希格斯玻色子质量的辐射修正仍有相当大的相对占比，因此需要很好地控制高阶项来减小不确定性。相关计算已在文献 [139] 中综述。领头修正由顶夸克的汤川耦合控制:

$$\Delta m_{h^0}^2 \approx \frac{3m_t^4}{4\pi^2 v^2}\left[\ln\frac{M_{\tilde{T}}^2}{m_t^2} + \frac{X_t^2}{M_{\tilde{T}}^2} - \frac{X_t^4}{12M_{\tilde{T}}^4}\right] \tag{38}$$

where $M_{\tilde{T}} \sim M_{\text{soft}} \gg m_t$ represents the average mass of the scalar partners of the top, while $X_t \sim M_{\text{soft}}$ parameterizes their mixing, which is induced by electroweak symmetry breaking. The first term corresponds to an ultraviolet logarithm and shows that, in the presence of a hierarchy between the standard and the supersymmetric sectors, the expansion parameter in the perturbative series is not simply $\frac{3m_t^2}{4\pi^2 v^2}$ but $\frac{3m_t^2}{4\pi^2 v^2}\ln\frac{M_{\text{soft}}^2}{m_t^2}$. This implies the continuing emergence of large effects at higher orders, e.g., of the form $\frac{\alpha_S m_t^2}{\pi^3 v^2}\ln^2\frac{M_{\text{soft}}^2}{m_t^2}$ and $\frac{m_t^4}{\pi^4 v^4}\ln^2\frac{M_{\text{soft}}^2}{m_t^2}$ at two-loop. These ultraviolet logarithms can be resummed using the effective field theory techniques, the impact of this resummation being already substantial for $M_{\text{soft}} \approx 1\text{TeV}$. Non-logarithmic and electroweak corrections are naturally also needed for precision predictions. As such, the higher-order uncertainty on the Higgs mass prediction may still remain above 1GeV in scenarios with TeV-scale supersymmetric sectors.

其中 $M_{\tilde{T}} \sim M_{\text{soft}} \gg m_t$ 代表顶夸克标量伙伴的平均质量，$X_t \sim M_{\text{soft}}$ 表征由电弱对称破缺诱发的它们的混合。第一项对应紫外对数，表明如果标准模型能区与超对称能区之间存在能标层级，微扰级数中的展开参数并非单纯的 $\frac{3m_t^2}{4\pi^2 v^2}$ ，而是 $\frac{3m_t^2}{4\pi^2 v^2}\ln\frac{M_{\text{soft}}^2}{m_t^2}$ 。这意味着大效应会持续出现在更高阶，例如两圈阶的 $\frac{\alpha_S m_t^2}{\pi^3 v^2}\ln^2\frac{M_{\text{soft}}^2}{m_t^2}$ 和 $\frac{m_t^4}{\pi^4 v^4}\ln^2\frac{M_{\text{soft}}^2}{m_t^2}$ 形式的项。这些紫外对数可以通过有效场论方法重求和，对于 $M_{\text{soft}} \approx 1\text{TeV}$ ，这种重求和的影响已经十分显著。为了得到精确预言，自然也需要计入非对数修正与电弱修正。因此，在 TeV 标度超对称能区的情景中，希格斯质量预言的高阶不确定性仍可能大于 1GeV 。

Of course, the Higgs mass is only one electroweak observable among many. For a decoupling scenario, the Higgs couplings are expected to be Standard Model-like, which is consistent with the current experimental status: narrower determinations in the future may place indirect constraints on the non-standard spectrum. Leaving the Higgs sector, the relations among the electroweak parameters - fine structure constant, Fermi constant, W - and Z -masses, and the gauge couplings to fermions - constrain the non-standard spectrum. The reader may refer to [92, 93] for analyses in the MSSM. Nevertheless, for models with only doublet (and singlet) vevs and a heavy (TeV-like) new physics sector, such observables are expected to be in good agreement with the Standard Model.

当然，希格斯质量只是众多电弱观测值中的一个。在退耦情景下，希格斯耦合预期与标准模型一致，这符合当前的实验状况: 未来更精确的测量可能会对非标准谱给出间接约束。抛开希格斯能区不谈，电弱参数之间的关系——精细结构常数、费米常数、W 、Z 质量，以及规范玻色子与费米子的耦合——都对非标准谱形成约束。读者可以参考 [92, 93] 了解 MSSM 中的相关分析。不过，对于仅存在二重态 (和单态) 真空期望值且新物理能区为重 (TeV 量级) 的模型，这类观测量预期都与标准模型符合得很好。

Another precision observable of great interest (though only loosely related to electroweak symmetry breaking) is the anomalous magnetic moment of the muon, where a historically durable deviation with the Standard Model prediction was recently confirmed by the Fermilab Muon $g-2$ experiment. The discrepancy is of comparable magnitude with the standard electroweak contributions to this observable and could thus hint at new physics effects at comparable scales. The status of this observable in the MSSM is reviewed in [145]: gauginos, higgsinos, and/or sleptons are then expected well below the TeV scale in order to account for the measured anomaly, to which these particles contribute already at one-loop order.

另一个备受关注的精确观测量 (尽管仅与电弱对称性破缺松散相关) 是缪子的反常磁矩，历史上长期存在的与标准模型预测的偏差最近已被费米实验室缪子 $g-2$ 实验证实。该偏差的大小与标准电弱理论对这个观测量的贡献幅度相当，因此可能暗示在相近能标处存在新物理效应。[145] 综述了该观测量在 MSSM 中的情况: 要解释测量得到的反常，胶微子、希格斯微子和/或 sleptons(sleptons 即超轻子) 应该存在于远低于 TeV 的能标，这些粒子在单圈阶就会对该反常产生贡献。

Let us finally comment on the question of the stability of the electroweak symmetry breaking vacuum. Indeed, the scalar potential of the MSSM is not restricted to Eq. (36), but also includes squarks and sleptons. Vacuum expectation values of charged or colored fields may lead, for a given choice of Lagrangian parameters, to minima deeper than the electroweak symmetry breaking one, thus endangering the phenomenological consistency of an expansion in the vicinity of that specific minimum - meta-stability may still legitimize the point in parameter space. Approximate analytical constraints on the parameters were derived at tree level early on. A full analysis of vacuum stability is nevertheless considerably more involved, due to the large dimensionality of the space of scalar fields, and even more so when including radiative corrections. We refer the reader to [95] for a recent discussion and a list of references.

最后我们来讨论电弱对称性破缺真空的稳定性问题。事实上，MSSM 的标量势并不局限于式 (36)，还包含 squarks(squarks 即超夸克) 和超轻子。对于拉格朗日参数的特定选择，带电或带色场的真空期望值可能会产生比电弱对称性破缺真空更深的最小值，从而危及在该特定最小值附近展开的唯象一致性——亚稳定性仍然可以使该参数空间点合法化。早期已经在树图水平得到了参数的近似解析约束。然而，由于标量场空间维度很高，真空稳定性的完整分析要复杂得多，纳入辐射修正后就更是如此。读者可以参考 [95] 了解最新的讨论和参考文献列表。

The Supersymmetry Flavor Problem

超对称味问题

The flavor structure of the Standard Model is particularly simple in that only the misalignment between the Yukawa coupling matrices in the quark sector allows for flavor transitions governed by the Cabibbo-Kobayashi-Maskawa matrix V^{CKM} and mediated by the electroweak charged current. This feature is lost when considering a supersymmetric extension of the Standard Model, because the soft breaking terms in the scalar potential introduce new sources of flavor violation that are potentially independent from the Yukawa couplings: these are the quadratic mass terms $m^2_{\tilde{F}} f f'$ $\left(\tilde{F} = \tilde{Q}, \tilde{U}^c, \tilde{D}^c, \tilde{L}, \tilde{E}^c\right)$ and the trilinear couplings $(A)^{ff'}_{u,d,e}$. Even though an alignment of the soft terms with the Yukawa structure would be engineered at, e.g., m_p by the supersymmetry breaking mechanism, this feature is not preserved by the renormalization group evolution, so that the sfermion couplings at M_{soft} would still be misaligned with the fermion couplings.

标准模型的味结构格外简单: 只有夸克区汤川耦合矩阵之间的失准, 才会允许卡比博-小林- Maskawa 矩阵 V^{CKM} 描述、电弱带电流介导的味跃迁。这一特性在考虑标准模型的超对称推广时不复存在，因为标量势中的软破缺项引入了潜在独立于汤川耦合的新味违逆来源: 即二次质量项 $m^2_{\tilde{F}} f f'$ $\left(\tilde{F} = \tilde{Q}, \tilde{U}^c, \tilde{D}^c, \tilde{L}, \tilde{E}^c\right)$ 和三线性耦合项 $(A)^{ff'}_{u,d,e}$。即便超对称破缺机制能让软项在例如 m_p 能标处与汤川结构对齐，这一特性也无法被重整化群演化保留，因此 M_{soft} 能标处的标量费米子耦合仍会与费米子耦合失准。

Flavor violation in the quark sector is tested with remarkable accuracy in low-energy transitions such as meson oscillations, e.g., $K{-}\bar{K}, B_d{-}\bar{B}_d$, and $B_s{-}\bar{B}_s$, or rare meson decays, e.g., $K \to \pi\upsilon\bar{\upsilon}, \bar{B} \to X_s\gamma, B_s \to \mu^+\mu^-$, and $B^+ \to \tau^+\upsilon_\tau$. Flavor violation in the squark sector then typically contributes at the loop level, with squarks and gauginos/higgsinos running in the loop. In particular, the neutral squark interaction with gluinos may induce a flavor transition in association with the strong coupling. If the corresponding flavor structure is completely unrelated to V^{CKM}, experimental bounds on, e.g., meson oscillation parameters constrain the flavor-violating spectrum up to scales typically reaching beyond 100 TeV [100]. Slightly milder limits (the strong coupling does not contribute at leading order) emerge in the lepton sector from observables such as $\mu \to e\gamma$.

夸克区的味违逆已经在低能跃迁过程中得到极高精度的检验，例如介子振荡 (如 $K - \bar{K}, B_d - \bar{B}_d$ 、$B_s - \bar{B}_s$) 或稀有介子衰变 (如 $K \to \pi\upsilon\bar{\upsilon}, \bar{B} \to X_s\gamma, B_s \to \mu^+\mu^-$ 、$B^+ \to \tau^+\upsilon_\tau$)。标夸克区的味违逆通常在圈图水平贡献，圈中运行的是标夸克与 gauginos/higgsinos。特别是，中性标夸克与胶微子的相互作用可在强耦合作用下诱发味跃迁。如果对应的味结构与 V^{CKM} 完全无关，实验对介子振荡参数等物理量的限制，就会约束味破缺能谱，范围通常超过 100 TeV[100]。轻子区从 $\mu \to e\gamma$ 这类观测量得到的限制稍松 (强耦合不领头阶贡献)。

As a consequence of the tight pattern of flavor violation from an experimental perspective, globally consistent with a Standard Model interpretation, one must either renounce new physics close to the TeV scale and set $M_{\text{soft}} \geq 100\text{TeV}$ or assume that the flavor imprint of the Yukawa couplings also applies to the soft supersymmetry breaking terms at $M_{\text{soft}} \approx 1\text{TeV}$. This latter phenomenological hypothesis is known as minimal flavor violation - see [100] and references therein. In its simplest application, the squark and slepton sectors are exactly aligned with their fermionic counterparts, so that their mass matrices are block-diagonal in the basis of SM flavors and V^{CKM} is the only source of flavor violation in their charged interactions; flavor transitions can no longer be induced by neutral mediators such as the gluino. Still, flavor observables continue to constrain new physics effects. The latter, now following the V^{CKM} structure, are mediated by loops involving charged Higgs and quarks, or charged gauginos/higgsinos and squarks, resulting in limits on the masses of non-standard particles in the TeV range. Small deviations from a strict alignment can of course be considered as well.

从实验角度来看，味违逆的整体模式严格符合标准模型解释，因此我们要么必须放弃 TeV 能标附近存在新物理，将 $M_{\text{soft}} \geq 100\text{TeV}$ 设定在更高能标，要么假设汤川耦合的味印记同样适用于 $M_{\text{soft}} \approx 1\text{TeV}$ 处的软超对称破缺项。后一种唯象假设被称为最小味违反——参见文献 [100] 及其中引用。在最简单的情况中，标夸克和标轻子区与它们对应的费米子区精确对齐，因此它们的质量矩阵在标准模型味基矢下是分块对角的，且 V^{CKM} 是其带相互作用中唯一的味违逆来源；味跃迁无法再被胶微子这类中性媒介子诱发。即便如此，味观测量仍然会约束新物理效应。此时新物理效应遵循 V^{CKM} 结构，通过包含带电希格斯与夸克、或带电 gauginos/higgsinos 与标夸克的圈图介导，得到 TeV 范围内非标准粒子的质量限制。当然，也可以考虑严格对齐下的微小偏离。

This phenomenological picture of alignment at low energy is not really satisfactory from the perspective of model building and calls for interpretations. The most popular strategy consists in producing the soft supersymmetry breaking terms via the gauge mediation mechanism [80], with comparatively light mediators (”messengers”) at about 10-100 TeV: gauge interactions would then act in accordance with the existing Yukawa structure in the observable sector, and the low scale of the mediation would forbid any significant deviation from alignment to develop via running effects. The situation is more critical in gravity mediation, where the generation of the soft terms involves physics at m_p : a solution to the flavor problem then implies to ”guess” the physics behind the flavor structure. A possible approach consists in regarding all flavor structures (Yukawas, soft terms) as spurions, i.e., as relics of fields (”flavons”) governing the dynamics behind the breaking of the flavor symmetry group. The prototype of such constructions is the Froggatt-Nielsen model, using a horizontal $U(1)$ symmetry [72, 88] .

这种低能标对齐的唯象图景从模型构建的角度来看并不十分理想，需要给出合理解释。最主流的解决方案是通过规范中介机制产生软超对称破缺项 [80]，该机制引入的媒介子 (“信使”) 质量相对较轻，约为 10-100 TeV: 规范相互作用会与可观测区已有的汤川结构一致，而中介的低能标会禁止通过跑动效应产生任何偏离对齐的显著偏移。引力中介的情况则更为严峻，在该框架下软项的生成涉及 m_p 处的物理: 因此要解决味问题就必须“推测”味结构背后的物理。一种可行思路是将所有味结构 (汤川耦合、软项) 都视为赝场，即它们是支配味对称群破缺后动力学的场 (“味子”) 的遗留产物。这类构造的原型是弗罗加特-尼尔森模型，该模型采用了水平 $U(1)$ 对称性 [72, 88] 。

CP violation raises an issue comparable to that of flavor to the MSSM. Even under the assumption of a flavor-aligned sfermion structure, several phases of physical meaning (beyond that in V^{CKM}) are a priori allowed in the soft sector, in association with the gaugino mass terms $M_{1,2,3}$ and the trilinear couplings $A_{u,d,e}$. On the one hand, new sources of CP violation are desirable in order to account for the baryon-antibaryon asymmetry of the universe. On the other hand, CP-violating phases are tightly constrained by the absence of any experimental evidence for electric dipole moments in nuclei and atoms [49]. Constructions similar to those addressing the flavor problem can be designed [125].

CP 破缺给最小超对称标准模型带来了一个与味问题性质类似的问题。即使假设标量费米子结构满足味对齐, 在软区中除了 V^{CKM} 内的相位外, 还有多个有物理意义的相位是先验允许的, 它们与戈金诺质量项 $M_{1,2,3}$ 以及三线性耦合项 $A_{u,d,e}$ 相关。一方面，新的 CP 破缺源有助于解释宇宙的重子-反重子不对称性。另一方面，由于实验上从未在原子核和原子中观测到电偶极矩，CP 破缺相位受到了严格约束 [49]。我们可以构造出与解决味问题的方案类似的理论构造 [125]。

Dark Matter Phenomenology

暗物质唯象学

A distinctive feature of R -parity conserving supersymmetry is the stability of the lightest R -odd (i.e.,”supersymmetric”) particle: indeed, as there exists no lighter final state with R -parity -1 (by definition), this particle cannot decay if R -parity is conserved. Then, according to the usual understanding of the thermal history of the universe, a stable particle coupled to the Standard Model would leave thermal relics when it drops out of equilibrium in the cooling universe (”freeze-out”) [82]. As such, the lightest supersymmetric particle may contribute to the dark matter relic density. Given strong astrophysical constraints on the abundance of charged particles, the most promising candidates in the MSSM sector are the (neutral) scalar partners of the neutrinos (sneutrinos) and the neutralinos, i.e., the neutral (and uncolored) gauginos and higgsinos, which mix after electroweak symmetry breaking.

在满足 R -宇称守恒的超对称中，一个显著特征是最轻的 R -奇 (即“超对称”) 粒子具有稳定性: 实际上，根据定义，不存在质量更轻、R -宇称为-1 的末态，因此若 R -宇称守恒，该粒子无法发生衰变。根据目前对宇宙热历史的普遍认知，一个与标准模型耦合的稳定粒子，在冷却宇宙中脱离平衡态 (“退耦”) 后，会留下热遗迹 [82]。因此，最轻超对称粒子可以贡献暗物质遗迹密度。由于天体物理对带电粒子丰度给出了极强限制，在 MSSM 领域中，最有前景的候选者是中微子的 (中性) 标量伙伴 (中微子超对称伴子) 和中性微子——即电弱对称性破缺后发生混合的中性 (无颜色)gaugino 和希格斯微子。

All the MSSM dark matter candidates belonging to a non-trivial $SU(2)_L$ multiplet (winos, higgsinos, sneutrinos) tend to annihilate very efficiently in the early universe, so that they would leave negligible thermal relics unless their mass is in the TeV range or above. On the contrary, a singlet fermion such as the bino - or the singlino, i.e., the fermionic component of the singlet superfield, in the NMSSM - tends to lead to excessive relics if left to itself. An excess in dark matter is a priori more problematic than a shortage, as other sectors (beyond the MSSM) could be invoked as complementary (or essential) sources of dark matter. However, several mechanisms can be called upon to boost the annihilation cross-sections of singlet fermions: sizable mixing with the other electroweakly charged neutralinos; resonant annihilation in a Higgs (or Z) ”funnel”; presence of a comparatively light t -channel mediator, e.g., a slepton; and existence of a heavier but almost degenerate R -odd particle, still abundant at the moment of freeze-out and helping in the depletion of R -odd particles via ”co-annihilation” processes [83].

MSSM 中所有属于非平庸 $SU(2)_L$ 多重态的暗物质候选者 (wino、希格斯微子、中微子超对称伴子) 在早期宇宙中倾向于发生极高效率的湮灭，因此除非它们的质量处于 TeV 量级或更高，否则留下的热遗迹可以忽略不计。相反，若存在单态费米子，比如 NMSSM 中的 bino(即单态超场的费米分量) 或 singlino，若仅靠其自身作用，往往会产生过多的遗迹。原则上，暗物质过剩比暗物质不足问题更严重，因为其他领域 (MSSM 之外) 可以作为补充 (或必要的) 暗物质来源。不过，可以通过多种机制提高单态费米子的湮灭截面: 与其他电弱荷电中性微子发生显著混合；在希格斯 (或 Z)“漏斗区”发生共振湮灭；存在质量相对较轻的 t 道媒介子 (例如超轻子)；以及存在质量更重但几乎简并的 R -奇粒子，这种粒子在退耦时刻仍然丰度很高，可以通过“共同湮灭”过程帮助消耗 R -奇粒子 [83]。

The wish to explain all of the observed dark matter with a comparatively light MSSM candidate, as well as theoretical prejudice on the spectra, made the case of a bino-dominated dark matter, dependent on the previous mechanisms, a popular one in the latest few decades. Nevertheless, one should be aware of the numerous caveats behind this hypothesis. First, the lightest R -odd particle of the MSSM sector need not be the lightest R -odd particle in absolute: obvious competitors would be a lighter gravitino, typically in a gauge mediation context, or any other exotic particle, e.g., an axino, partner of an axion introduced to address the strong CP problem. Then, small R -parity-violating effects, negligible or not in collider physics, may render the lightest supersymmetric particle unstable on cosmological scales. Finally, non-thermal effects or as yet unknown shortcomings in the formulation of standard cosmology may ruin the thermal picture. Below, we will forget about these warnings and focus on the phenomenology of a weakly interacting thermal explanation of cold dark matter.

过去几十年来，希望用质量相对较轻的 MSSM 候选者解释全部观测到的暗物质，加上对能谱的理论偏好，使得依赖上述机制的 bino 主导暗物质成为非常流行的假说。不过我们需要注意该假说背后存在诸多需要说明的问题。首先，MSSM 领域的最轻 R -奇粒子不一定是绝对意义上的最轻 R -奇粒子: 明显的竞争者可以是质量更轻的引力微子 (通常出现在规范媒介框架下)，或是任何其他外来粒子，例如为解决强 CP 问题引入的轴子的超对称伙伴轴微子。其次，哪怕是很小的 R -宇称破缺效应 (无论在对撞机物理中是否可忽略)，都可能让最轻超对称粒子在宇宙学尺度上不稳定。最后，非热效应或是标准宇宙学表述中尚未发现的缺陷，都可能破坏热退耦图像。在下文中，我们暂不考虑这些问题，聚焦于弱相互作用热暖冷暗物质的唯象学。

We have already copiously discussed the dark matter relic density: this quantity is extracted from the measured properties of the cosmic microwave background, corresponding to the radiation trace of the epoch of atom formation (recombination). Its evolution up to this day has been affected through gravitational interactions by the matter/energy content of the universe. Other evidence for dark matter, such as the rotation curves of galaxies or gravitational lensing, can convince us of the existence of dark matter in the current universe and more specifically in the solar system. Two main experimental strategies are pursued in the attempt at detecting its by-definition faint interactions with Standard Model matter. The first one, ”direct detection,” consists in searching for the recoil of heavy nuclei in elastic collisions with dark matter particles. Such experiments steadily progress in covering the available plane mass vs. cross-section down to the ”neutrino floor,” at which the competition of neutrinos (of sun, earth, or cosmic origin) shall raise a challenge for further investigation. The alternative strategy, ”indirect detection,” looks for dark matter annihilation currently taking place in regions of space where dark matter is expected to be dense, inside massive bodies or near the galactic center. Typical signals would be energetic neutrinos or gamma rays, detectable either in earth-, air-, or space-based experiments. In both direct and indirect detection strategies, the identification of a signal involving dark matter necessarily depends on the astrophysical modelization of dark matter densities and fluxes in the investigated regions, so that the consequences for particle physics might be tempered. To this day, no robust discovery of a dark matter signal in direct or indirect detection has been reported.

我们已经对暗物质遗迹密度做了大量讨论: 该量是从宇宙微波背景的测量性质中提取得到的，对应原子形成 (复合) 时期的辐射痕迹。从大爆炸到今日，宇宙的物质/能量含量通过引力相互作用影响了暗密度的演化。其他暗物质证据，比如星系旋转曲线或引力透镜，能让我们确信暗物质在当今宇宙中、特别是太阳系中的确存在。目前有两种主要实验策略来探测暗物质和标准模型物质本就微弱的相互作用。第一种是「直接探测」，即寻找暗物质粒子弹性碰撞后重原子核的反冲信号。这类实验在覆盖质量-截面参数空间的过程中稳步推进，已经深入到「中微子底」，在此处太阳、地球或宇宙起源的中微子本底会对进一步探测造成阻碍。另一种策略是「间接探测」，寻找暗物质在空间中预计密度较高的区域 (大质量天体内部或银心附近) 当前发生湮灭产生的信号。典型信号是高能中微子或伽马射线，可通过地面、空中或空间实验探测到。无论是直接探测还是间接探测，识别暗物质信号都必然依赖对研究区域内暗物质密度和流强的天体物理建模, 因此其对粒子物理的推论结论会受到不确定性影响。直到今天，尚未有直接探测或间接探测发现可靠暗物质信号的报道。

Independent of whether the lightest R -odd particle of the MSSM represents a sizable component of dark matter, one can also attempt to produce it at colliders from Standard Model matter, which leads us to the collider phenomenology. A more detailed overview of supersymmetric dark matter may be found in, e.g., [21].

不管 MSSM 中最轻的 R -奇粒子是否占暗物质的相当大比例，我们都可以尝试在对撞机上通过标准模型物质产生它, 这就引出了对撞机唯象学。更详细的超对称暗 matter 综述可以参考例如文献 [21]。

Collider Phenomenology

对撞机唯象学

The traditional investigation path in experimental particle physics consists in accelerating electrons and/or protons to very high energies before colliding them: the high kinetic energy of the projectiles may then convert into interactions involving very massive particles, thus providing access to physical effects beyond electromagnetism and nuclear forces. In this fashion, new physics resonances could be directly produced, provided the kinematical threshold is reached and the cross-section × integrated luminosity is sufficiently large to make such events probable enough in collisions and detectable with the available experimental sensitivity. An alternative strategy, which we mentioned in previous subsections, consists in precisely measuring the properties of known particle to attempt and detect effects ascribable to physics beyond the Standard Model. We here focus on the direct production approach.

实验粒子物理学的传统研究路径是将电子和/或质子加速至极高能量后让它们对撞: 入射粒子的高动能可转化为包含大质量粒子的相互作用，从而让我们得以研究电磁力和核力之外的物理效应。只要达到运动学阈值，且积分亮度的截面 × 足够大，使得这类事件在对撞中足够大概率发生，且可在现有实验灵敏度下被探测到，就能通过这种方式直接产生新物理共振态。另一种我们在前几小节提到过的研究策略，是精确测量已知粒子的性质，尝试探测可归属于超出标准模型物理的效应。本文我们聚焦于直接产生这一方向。

Under the assumption of R -parity conservation, one expects the supersymmetric particles to be produced in pairs in collisions of standard matter. The resonances may then either decay to a lighter R -odd particle through radiation of standard particles or escape the detector if they are stable or long-lived. For ”usual”

MSSM spectra, the strongly or electroweakly interacting R -odd particles decay promptly, via a cascade, down to the lightest supersymmetric particle. In some cases, however, e.g., if the decay has very little available phase space, or if couplings to the lightest supersymmetric particle are feeble (or suppressed by the recourse to, e.g., very massive mediators), a R -odd particle may be long-lived at the scales of the collider experiment (where large boost factors can substantially lengthen the apparent lifetime). The most studied scenario is that of a prompt decay into a stable and massive neutralino, which escapes the experiment without being detected, similar to a neutrino, resulting in a sizable drain of energy and momentum in the apparent balance of the transition. This ”missing energy” signature underlies the classical strategy for searches of supersymmetric particles at colliders.

在 R 宇称守恒的假设下，人们预期超对称粒子会在标准物质的对撞中成对产生。产生的共振态要么通过辐射标准粒子衰变到更轻的 R 奇粒子，若自身稳定或长寿命则会直接逃逸出探测器。对于“常规”的最小超对称标准模型能谱，强相互作用或电弱相互作用的 R 奇粒子会通过级联迅速衰变，直到变成最轻超对称粒子。但在某些情况下，例如衰变可用的相空间极小，或是与最轻超对称粒子的耦合极弱 (或需要借助极重媒介子而被压低)，R 奇粒子在对撞机实验的尺度下可能是长寿命的 (大洛伦兹 boost 因子会显著延长其表观寿命)。被研究最多的场景是粒子迅速衰变为稳定大质量中性微子，中性微子类似中微子，不会被探测到就逃逸出实验装置，导致反应的表观能动量平衡中出现显著的能量动量缺失。这种“缺失能量”信号是对撞机上搜寻超对称粒子经典方案的核心。

The production of supersymmetric pairs typically occurs through s -channel exchange of a gauge boson or t - and u -channel exchanges of a R -odd particle. At lepton colliders, the production process is electroweak, while it can be strong or electroweak at hadron colliders. The expected final states typically involve missing energy, plus jets (strongly interacting matter), plus leptons. A skillful prescription of cuts is usually necessary to distinguish such final states from the Standard Model background, for instance, QCD processes at a hadron collider, with radiated W s and Z s producing the leptons and missing energy. The absence of discovery at the Large Electron-Positron Collider placed limits on the masses of the supersymmetric particles in the 100 GeV range. Similarly, the Large Hadron Collider has been excluding vast areas of the parameter space available to R -odd particles, especially those that are produced in strong processes (squarks, gluinos): a typical lower bound on their mass would be one to a few TeV. For uncolored particles (sleptons, gauginos, higgsinos), the smaller (electroweak) production cross-sections result in milder limits, not exceeding a few 100 GeV. Final states with multiple leptons (plus missing energy) are the usual targets in this case. Of course, such experimental constraints are almost never generic, but generally apply to a specific type of spectra, so that their transposition to different scenarios usually requires the recourse to extrapolations or estimates (”recast”).

超对称粒子对的产生通常通过规范玻色子的 s 道交换，以及 R 奇粒子的 t 道和 u 道交换发生。在轻子对撞机中，产生过程是电弱相互作用过程，而在强子对撞机中，产生过程可以是强相互作用或电弱相互作用。预期的末态通常包含缺失能量，加上喷注 (强相互作用物质)，再加上轻子。通常需要设计巧妙的 cuts 方案才能将这类末态与标准模型本底区分开，例如强子对撞机中的量子色动力学过程，辐射产生的 W s 和 Z s 会给出轻子和缺失能量。大型正负电子对撞机未发现超对称粒子，已将超对称粒子的质量限制在 100 GeV 量级。类似地，大型强子对撞机已经排除了 R 奇粒子参数空间的大片区域，尤其是通过强相互作用过程产生的粒子 (squark、胶微子): 它们质量的典型下限为 1 TeV 到几 TeV。对于非彩色粒子 (sleptons、gauginos、希格斯微子)，由于 (电弱) 产生截面更小，得到的限制更宽松，不超过几百 GeV。这种情况下，多个轻子 (加缺失能量) 的末态是通常的搜寻目标。当然，这类实验约束几乎从来都不是普适的，通常只适用于特定类型的谱，因此要将它们推广到不同场景，通常需要外推或估计 (即“重构”)。

More exotic signatures are also looked for. For instance, long-lived particles decaying on length scales comparable to the detector size may produce leptons or jets with a point of origin distinct from the interaction point ("displaced leptons" and "displaced vertices"). A long-lived (or stable) charged particle could leave identifiable tracks. For larger lifetimes, the deployment of detectors placed at a few 10-100 meters from the interaction points has been planned at the Large Hadron Collider. However, in the absence of distinctive characteristics such as a large missing energy or the existence of long-lived heavy particles, the identification of new resonances at high-energy colliders might prove difficult in general; such a scenario would nevertheless be regarded as highly exotic. Finally, reconstructing the nature of the hypothetically discovered new fields and their classification in a supersymmetric spectrum would call for a long-term, as yet unforeseeable, effort.

人们也在搜寻更多奇特信号。例如，衰变长度尺度和探测器尺寸相当的长寿命粒子，可能产生起源点不同于相互作用点的轻子或喷注，即“位移轻子”和“位移顶点”。长寿命 (或稳定) 带电粒子会留下可识别的径迹。对于寿命更长的粒子，大型强子对撞机已经规划在距离相互作用点几十到一百米处放置探测器。不过，如果没有大缺失能量或长寿命重粒子这类显著特征，在高能对撞机上识别新共振态总体来说会十分困难；但这类场景本身也被认为非常奇特。最后，若假设发现了新场，要重构新场的性质并将其归入超对称能谱，还需要长期的、目前无法预估的投入。

The extended Higgs sector of supersymmetric extensions of the Standard Model offers another direction for investigations. The heavy doublet states H^0, A^0 , and $H^\pm$ are R -even, hence need not be produced in pairs. Typical production modes at the Large Hadron Collider are comparable to those of the Standard Model Higgs boson and involve gluon-gluon fusion, (electroweak) vector boson fusion, and assisted production with bottom and/or top quarks. At an e^+e^- collider, these heavy Higgs bosons would have to be produced in pairs or in association with an h^0 , from a Z -boson exchange in the s -channel. The dominant decay channels involve fermion pairs, with typical searches in $\tau^+\tau^-, \mu^+\mu^-, b\bar{b}$, or $t\bar{t}$ ($\tau\bar{\upsilon}_\tau$ or $b\bar{t}$ for H^-). Bosonic decay channels are suppressed in the MSSM, although $H^0 \to h^0 h^0$ may be detectable, provided H^0 is light enough (though with mass above threshold). In the NMSSM, however, Higgs-to-Higgs cascade decays involving singlet states may be dominant and overshadow the fermionic modes: these processes are also actively looked for.

标准模型的超对称扩展中，扩充后的希格斯区提供了另一个研究方向。重二重态 H^0, A^0 和 $H^\pm$ 为 R -正，因此不需要成对产生。大型强子对撞机上典型的产生模式与标准模型希格斯玻色子类似，包括胶子-胶子融合、(电弱) 矢量玻色子融合，以及伴随底夸克和/或顶夸克产生。在 e^+e^- 对撞机上，这些重希格斯玻色子必须成对产生，或是伴随 h^0 产生，源于 s 道中的 Z 玻色子交换。主要衰变道为费米子对，典型的搜索方向是 $\tau^+\tau^-, \mu^+\mu^-, b\bar{b}$ 、$t\bar{t}$ ($\tau\bar{\upsilon}_\tau$ 或 $b\bar{t}$ (针对 H^-)。在最小超对称标准模型中，玻色衰变道被压低，不过只要 H^0 足够轻 (质量仍高于产生阈)，$H^0 \to h^0 h^0$ 仍然可以被探测到。但在次最小超对称标准模型 (NMSSM) 中，包含单态的希格斯级联衰变可能占主导地位，掩盖费米衰变模式: 这些过程也在积极搜寻中。

A short summary of the search for supersymmetry at the Large Hadron Collider can be found in [19].

大型强子对撞机搜寻超对称性的简短综述可见文献 [19]。

Supersymmetric Grand Unification

超对称大统一

The MSSM and its singlet extension (NMSSM) are the simplest models fulfilling a softly broken supersymmetric embedding of the Standard Model. However, about any idea in non-supersymmetric particle physics, e.g., axions, vector quarks, and right-handed neutrinos, can be transposed to the supersymmetry/supergravity breaking framework of section ”Supergravity Breaking.” Here, we shall provide a few insights concerning Grand Unification in this context. Two coincidences of the Standard Model (or the MSSM) particularly motivate the idea of a unification of gauge interactions within a less disparate group. The first one is the cancellation of the chiral anomalies, accidentally resulting from the matter content. The second one is the quantization of the electric charge, i.e., the fact that all elementary particles take charges that are multiple of 1/3 : the hypercharge being a $U(1)$ symmetry, there is no deep reason why the various hypercharges in the Standard Model should appear in rational proportions. Reciprocally, the protection of radiative corrections by supersymmetry benefits grand unified constructions through the stabilization of scale hierarchies and the reduced risk of encountering a non-perturbative regime (Landau pole). Finally, we emphasize that the apparent convergence of the Standard Model gauge couplings at $M_{\mathrm{GUT}} \approx 10^{16}$GeV in the MSSM favors a one-step unification ($SU(5)$, $SO(10)$, etc.), with a ”Grand Desert” between the electroweak and Grand Unification scales, rather than the multistep path of left-right symmetry and Pati-Salam. For reviews of the group theoretical concepts involved in Grand Unification, we refer the reader to (e.g.) [18, 131, 138, 160] and to [115, 116, 129] concerning their application in a supersymmetric context.

MSSM 及其单态扩展 (NMSSM) 是满足标准模型软破缺超对称嵌入的最简模型。但非超对称粒子物理中的几乎所有思路，例如轴子、矢量夸克、右手中微子，都可以移植到“超引力破缺”小节的超对称/超引力破缺框架中。本文将在此框架下分享一些关于大统一的见解。标准模型 (或 MSSM) 的两个巧合特别推动了规范相互作用统一到更小差异群的想法: 第一个是手征反常恰好因物质组分抵消，第二个是电荷量子化——即所有基本粒子的电荷都是 1/3 的倍数: 超荷是一种 $U(1)$ 对称性，没有深层原因能解释标准模型中各类超荷为何呈现有理比例。反过来说，超对称对辐射修正的保护作用能稳定标度层级，降低遇到非微扰区域 (朗道极点) 的风险，对大统一构造十分有利。最后我们要强调，MSSM 中标准模型规范耦合在 $M_{\mathrm{GUT}} \approx 10^{16}$GeV 处明显收敛，支持存在“大沙漠”(电弱标度与大统一标度之间无新物理) 的一步统一 ($SU(5)$, $SO(10)$ 等方案，而非左右对称和帕蒂-萨拉姆的多步统一路径。关于大统一涉及的群论概念综述，读者可参考 (例如) 文献 [18, 131, 138, 160]，其超对称框架下的应用可参考 [115, 116, 129]。

The simplest embedding of G_{SM} in a compact simple Lie group employs an $SU(5)$ gauge symmetry, thus involving no reduction of rank in spontaneous symmetry breaking from $SU(5)$ to G_{SM} . One may view the first three rows of the fundamental $SU(5)$ representation as transforming under the fundamental representation of $SU(3)_c$ and the two last, under the fundamental representation of $SU(2)_L$. Finally, the generator $\mathbf{t}^{24} = \sqrt{\frac{3}{5}}\,\mathrm{diag}\left(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{1}{2}, \frac{1}{2}\right)$ commutes with all $SU(3)_c$ and $SU(2)_L$ generators and can be identified, up to a proportionality constant, to the hypercharge: $\mathbf{Y} = C_{24}^{Y}\mathbf{t^{24}}$. The conventional normalization of the hypercharge leads to $C_{24}^{Y} = \sqrt{\frac{5}{3}}$ (because all generators in the fundamental representation satisfy $\mathrm{Tr}\left(\mathbf{t}^a\mathbf{t}^b\right) = \delta^{ab}/2$). This connection explains the quantization of the electric charge. Then, studying the branching rules of $SU(5) \supset G_{\mathrm{SM}}$, one observes

将 G_{SM} 嵌入紧致单李群的最简方式采用了 $SU(5)$ 规范对称性，因此从 $SU(5)$ 自发对称破缺到 G_{SM} 时不会降低秩。可以认为，基础 $SU(5)$ 表示的前三行按 $SU(3)_c$ 的基础表示变换，后两行按 $SU(2)_L$ 的基础表示变换。最后，生成元 $\mathbf{t}^{24} = \sqrt{\frac{3}{5}}\,\mathrm{diag}\left(-\frac{1}{3},-\frac{1}{3},-\frac{1}{3},\frac{1}{2},\frac{1}{2}\right)$ 与所有 $SU(3)_c$ 和 $SU(2)_L$ 生成元对易，在相差一个比例常数的前提下可对应为超荷: $\mathbf{Y} = C_{24}^{Y}\mathbf{t^{24}}$ 。超荷的常规归一化给出 $C_{24}^{Y} = \sqrt{\frac{5}{3}}$ (因为基础表示中所有生成元都满足 $\mathrm{Tr}\left(\mathbf{t}^a\mathbf{t}^b\right) = \delta^{ab}/2$)。这种关联解释了电荷量子化。随后，研究 $SU(5) \supset G_{\mathrm{SM}}$ 的分支规则可观察到

$$\bar{\underline{5}} = \left(\bar{3},1,\frac{1}{3}\right)\oplus\left(1,2,-\frac{1}{2}\right),\ \underline{5} = \left(3,1,-\frac{1}{3}\right)\oplus\left(1,2,\frac{1}{2}\right),$$

$$\mathbf{10} = \left(\bar{3},1,-\frac{2}{3}\right)\oplus(1,1,1)\oplus\left(3,2,\frac{1}{6}\right), \tag{39}$$

$$\mathbf{24} = (\mathbf{8},\mathbf{1},0)\oplus(\mathbf{1},\mathbf{3},0)\oplus(\mathbf{1},\mathbf{1},0)\oplus\left(\mathbf{3},\mathbf{2},-\frac{5}{6}\right)\oplus\left(\bar{\mathbf{3}},\mathbf{2},\frac{5}{6}\right).$$

Consequently, the chiral superfields containing the Standard Model fermions can be embedded in (three generations of) a $\bar{\mathbf{5}}_F^f \oplus \mathbf{10}_F^f$ representation of $SU(5)$ (see (31)), while $H_{U,D}$ are identified with the doublet components of $\underline{\mathbf{5}}_S \oplus \bar{\mathbf{5}}_S$ (see (32)). In the adjoint representation (24) of $SU(5)$, one finds an embedding for all the Standard Model gauge (super)fields, with the additional $\left(\underline{3},\underline{2},-\frac{5}{6}\right)\oplus\left(\underline{3},\underline{2},\frac{5}{6}\right)$ called lepto-quarks. The latter take mass at the scale of the breaking $SU(5) \to G_{\mathrm{SM}}$ and convey new physics effects, such as lepton and baryon number violation. These should remain small at the electroweak scale for a phenomenologically realistic model, so that the breaking scale needs satisfy $M_{\mathrm{GUT}} \gg M_Z$. In fact, M_{GUT} is most naturally chosen as the scale at which the standard gauge couplings converge, i.e., $M_{\mathrm{GUT}} \approx 10^{16}\mathrm{GeV}$ for a MSSM field content with $M_{\mathrm{soft}} \approx 1\mathrm{TeV}$. The field content described above does not allow the breaking $SU(5) \to G_{\mathrm{SM}}$ (i.e., would break G_{SM} simultaneously with $SU(5)$), so that, as generic in grand unified models, further fields belonging to representations of larger dimensions are needed: the simplest choice consists in introducing 24_S in the adjoint representation of $SU(5)$ and taking a vev proportional to $\mathbf{t^{24}}$. This field content is again (accidentally) anomaly-free.

因此，包含标准模型费米子的手征超场可以嵌入 $SU(5)$ 的 $\bar{\mathbf{5}}_F^f \oplus \mathbf{10}_F^f$ 表示 (三代) 中 (见 (31))，而 $H_{U,D}$ 对应 $\underline{\mathbf{5}}_S \oplus \bar{\mathbf{5}}_S$ 的二重态分量 (见 (32))。在 $SU(5)$ 的伴随表示 (24) 中，可以嵌入所有标准模型规范 (超) 场，还存在额外的 $\left(\underline{3},\underline{2},-\frac{5}{6}\right)\oplus\left(\underline{3},\underline{2},\frac{5}{6}\right)$ ，称为轻夸克。轻夸克在破缺能标 $SU(5) \to G_{\mathrm{SM}}$ 获得质量，并带来新物理效应，比如轻子数和重子数破坏。要得到唯象上现实的模型，这些效应在电弱能标必须保持很小，因此破缺能标需要满足 $M_{\mathrm{GUT}} \gg M_Z$ 。实际上，M_{GUT} 最自然地被选为标准规范耦合收敛的能标，即对于包含 $M_{\mathrm{soft}} \approx 1\mathrm{TeV}$ 的 MSSM 场内容，该能标为 $M_{\mathrm{GUT}} \approx 10^{16}\mathrm{GeV}$ 。上述场内容无法实现破缺 $SU(5) \to G_{\mathrm{SM}}$ (即会在破缺 $SU(5)$ 的同时破缺 G_{SM})，因此和大统一模型的一般情况一样，需要引入属于更大维表示的额外场: 最简单的选择是在 $SU(5)$ 的伴随表示中引入 24_S ，并取正比于 $\mathbf{t^{24}}$ 的真空期望值。该场内容再次 (偶然地) 没有反常。

We may now write the superpotential for the $SU(5)$ model thus designed:

现在我们可以写出这样构造的 $SU(5)$ 模型的超势:

$$W_{SU(5)} = \frac{m}{2}\,\mathrm{Tr}\,(\mathbf{24}_S)^2 + \frac{\lambda}{3}\,\mathrm{Tr}\,(\mathbf{24}_S)^3 + \mu\left(\bar{\mathbf{5}}_S\right)_i\left(\bar{\bar{\mathbf{5}}}_S\right)^i + \beta\left(\bar{\bar{\mathbf{5}}}_S\right)^i(\mathbf{24}_S)_i{}^j\left(\bar{\mathbf{5}}_S\right)_j$$

$$+Y_5\left(\bar{\mathbf{5}}_F\right)^i\left(\overline{\mathbf{10}}_F\right)_{ij}\left(\bar{\mathbf{5}}_S\right)^j+Y_{10}\varepsilon^{ijk\ell m}\left(\overline{\mathbf{10}}_F\right)_{ij}\left(\overline{\mathbf{10}}_F\right)_{k\ell}\left(\bar{\mathbf{5}}_S\right)_m \tag{40}$$

where the indices $i,\ldots,m$ correspond to the transformation under the (anti)fundamental $SU(5)$ (the generation indices are omitted) and $\varepsilon^{ijk\ell m}$ is the five-dimensional Levy-Civita symbol. The matter parity, i.e., invariance under $\bar{\mathbf{5}}_F,\mathbf{10}_F \rightarrow -\bar{\mathbf{5}}_F,-\mathbf{10}_F$, has been implicitly required. The $SU(5)$ -breaking minimum $<24_S>=\frac{m}{\lambda}\operatorname{diag}(2,2,2,-3,-3)$ does not break supersymmetry and is therefore degenerate with the $SU(5)$ -conserving one as long as supersymmetry breaking terms are not introduced. The symmetry breaking $G_{\mathrm{SM}}\rightarrow SU(3)_c\times U(1)_{\mathrm{em}}$ is achieved by the doublet vevs of $\underline{\mathbf{5}}_S\oplus\bar{\mathbf{5}}_S$. The two last terms in the first line of Eq. (40) generate an effective μ -term $\mu_{\text{eff}}=\mu-\frac{3m\beta}{\lambda}$ for $H_{U,D}$ (embedded in $\underline{\mathbf{5}}_S\oplus\bar{\mathbf{5}}_S$), while the supersymmetric mass associated with the color triplets $H_3,\bar{H}_3$ of $\underline{\mathbf{5}}_S\oplus\bar{\mathbf{5}}_S$ is $\mu+\frac{2m\beta}{\lambda}$. From the phenomenological perspective, it is necessary to impose the fine-tuning $|\mu_{\text{eff}}|\ll|\mu|,\left|\frac{3m\beta}{\lambda}\right|$, making the doublets light and the color triplets superheavy: while this requirement is non-natural, it remains technically natural once set, due to the protection by supersymmetry - contrary to the situation in non-supersymmetric $SU(5)$. Further model-building ingredients can also naturally protect the doublet mass term.

其中指标 $i,\ldots,m$ 对应 (反) 基础表示 $SU(5)$ 下的变换 (世代指标已省略)，$\varepsilon^{ijk\ell m}$ 是五维列维-奇维塔符号。此处已隐式要求满足物质宇称，即 $\bar{\mathbf{5}}_F,\mathbf{10}_F\rightarrow-\bar{\mathbf{5}}_F,-\mathbf{10}_F$ 变换下的不变性。破缺 $SU(5)$ 的极小点 $<24_S>=\frac{m}{\lambda}\operatorname{diag}(2,2,2,-3,-3)$ 不会破缺超对称，因此只要未引入超对称破缺项，它与守恒 $SU(5)$ 的极小点是简并的。对称性破缺 $G_{\mathrm{SM}}\rightarrow SU(3)_c\times U(1)_{\mathrm{em}}$ 由 $\underline{\mathbf{5}}_S\oplus\bar{\mathbf{5}}_S$ 的二重态真空期望值实现。式 (40) 第一行的最后两项为嵌入 $\underline{\mathbf{5}}_S\oplus\bar{\mathbf{5}}_S$ 中的 $H_{U,D}$ 生成了有效 μ 项 $\mu_{\text{eff}}=\mu-\frac{3m\beta}{\lambda}$，而与 $\underline{\mathbf{5}}_S\oplus\bar{\mathbf{5}}_S$ 的颜色三重态 $H_3,\bar{H}_3$ 对应的超对称质量为 $\mu+\frac{2m\beta}{\lambda}$。从唯象学角度看，必须引入微调 $|\mu_{\text{eff}}|\ll|\mu|,\left|\frac{3m\beta}{\lambda}\right|$，使得二重态质量很轻而颜色三重态质量超重: 尽管这一要求并不自然，但一旦设定，由于超对称的保护作用，它在技术上仍是自然的——这与非超对称 $SU(5)$ 中的情况不同。其他模型构建要素也可以自然地保护二重态质量项。

The terms in the second line of Eq. (40) are at the origin of the Yukawa couplings in the MSSM. Explicit decomposition immediately provides that Y_5 generates Yukawa couplings for the down-type quarks and leptons, while Y_{10} gives the up-type Yukawa. For the third generation, the resulting unification of down and lepton masses at the GUT scale $\left(m_b^{\mathrm{GUT}}=m_\tau^{\mathrm{GUT}}\right)$ roughly yields $m_b\sim 3m_\tau$ at low energy, which is in acceptable agreement with the measurements. This does not work for the lighter generations. Noting however that $\bar{\bar{\mathbf{5}}}\otimes\mathbf{10}=\underline{\mathbf{5}}\oplus\mathbf{45}$ and $\mathbf{10}\otimes\mathbf{10}=\underline{\bar{\mathbf{5}}}\oplus\overline{\mathbf{45}}\oplus\mathbf{50}$, one can introduce additional "Higgs" superfields in a $\mathbf{45}_S\oplus\overline{\mathbf{45}}_S$ representation, which allow for further terms of Yukawa type and contain doublets accepting a vev. The Clebsch-Gordan coefficients at M_{GUT} then produce $m_\mu^{\mathrm{GUT}}=3m_s^{\mathrm{GUT}}$ (for the second generation; in the absence of contributions from the $\underline{\mathbf{5}}_S\oplus\bar{\mathbf{5}}_S$ terms), resulting in the phenomenologically acceptable $m_\mu\sim m_s$. Thus, models involving both $\underline{\mathbf{5}}_S\oplus\bar{\mathbf{5}}_S$ and $\mathbf{45}_S\oplus\overline{\mathbf{45}}_S$ are potentially viable explanations of the unification of lepton and down-type masses. Similarly, the SU(5) symmetry implies the unification (at M_{GUT}) of numerous soft terms, such as the quadratic terms for all fields belonging to the same representation. As long as supersymmetry breaking is communicated to the observable sector via an $SU(5)$ singlet, the gaugino masses also unify (at M_{GUT}) $M_1^{\mathrm{GUT}}=M_2^{\mathrm{GUT}}=M_3^{\mathrm{GUT}}$, leading to the popular hierarchy $M_3\approx 3M_2\approx 6M_1$ at low energy.

(40) 式第二行中的项是 MSSM 中汤川耦合的起源。直接分解后可立即得到，Y_5 产生下型夸克和轻子的汤川耦合，而 Y_{10} 产生上型夸克的汤川耦合。对于第三代，GUT 能标 $\left(m_b^{\mathrm{GUT}} = m_\tau^{\mathrm{GUT}}\right)$ 下下型夸克与轻子质量的统一，在低能下大致给出 $m_b \sim 3m_\tau$ ，与测量结果符合得不错。但这一结论不适用于更轻的代。不过，注意到 $\overline{\underline{\mathbf{5}}} \otimes \mathbf{10} = \underline{\mathbf{5}} \oplus \mathbf{45}$ 和 $\mathbf{10} \otimes \mathbf{10} = \overline{\underline{\mathbf{5}}} \oplus \overline{\mathbf{45}} \oplus \mathbf{50}$ ，我们可以在 $\mathbf{45}_S \oplus \overline{\mathbf{45}}_S$ 表示中引入额外的“希格斯”超场，这些超场允许更多汤川型项，还包含可获得真空期望值的二重态。随后，M_{GUT} 处的 Clebsch-Gordan 系数会产生 $m_\mu^{\mathrm{GUT}} = 3m_s^{\mathrm{GUT}}$ (第二代的情况，此时不存在来自 $\underline{\mathbf{5}}_S \oplus \overline{\mathbf{5}}_S$ 项的贡献)，得到唯象上可接受的 $m_\mu \sim m_s$ 。因此，同时包含 $\underline{\mathbf{5}}_S \oplus \overline{\mathbf{5}}_S$ 和 $\mathbf{45}_S \oplus \overline{\mathbf{45}}_S$ 的模型是轻子与下型夸克质量统一的潜在可行解释。类似地，SU(5) 对称性意味着许多软项在 M_{GUT} 处统一，例如属于同一表示的所有场的二次项。只要超对称破缺通过 $SU(5)$ 单态传递到可观测扇区，戈迪诺质量也会在 M_{GUT} 处统一 $M_1^{\mathrm{GUT}} = M_2^{\mathrm{GUT}} = M_3^{\mathrm{GUT}}$ ，从而在低能下得到常用的层级关系 $M_3 \approx 3M_2 \approx 6M_1$ 。

Beyond $SU(5)$, the immediate benefit of considering unification groups of higher rank, such as $SO(10)$ or E_6 , rests with the automatic cancellation of chiral anomalies. The branching rules of $SO(10) \supset SU(5) \times U(1)$

除 $SU(5)$ 之外，考虑更高秩的统一群 (例如 $SO(10)$ 或 E_6) 的直接好处，是可以自动消除手征反常。$SO(10) \supset SU(5) \times U(1)$ 的分岐规则

$$16 = \left(\overline{5}, -3\right) \oplus (10, 1) \oplus (1, -5),\ 10 = \left(\underline{5}, 2\right) \oplus \left(\overline{5}, -2\right),$$

$$45 = (24, 0) \oplus (10, 4) \oplus \left(\overline{10}, -4\right) \oplus (1, 0) \tag{41}$$

$$\underline{\mathbf{120}} = \left(\underline{\mathbf{5}}, 2\right) \oplus \left(\overline{\underline{\mathbf{5}}}, -2\right) \oplus \left(\underline{\mathbf{10}}, -6\right) \oplus \left(\underline{\mathbf{45}}, 2\right) \oplus \left(\overline{\underline{\mathbf{45}}}, -2\right)$$

$$\underline{\mathbf{126}} = \left(\underline{\mathbf{1}}, -10\right) \oplus \left(\overline{\underline{\mathbf{5}}}, -2\right) \oplus \left(\underline{\mathbf{10}}, -6\right) \oplus \left(\overline{\underline{\mathbf{15}}}, 6\right) \oplus (\mathbf{45}, 2) \oplus \left(\overline{\underline{\mathbf{50}}}, -2\right)$$

suggest an embedding of all quark and lepton superfields within three generations of $\mathbf{16}_F^f$, also containing $SU(5)$ singlets amounting to right-handed neutrinos. Then, observing that $\underline{16} \otimes \underline{16} = \underline{10} \oplus \underline{120} \oplus \underline{126}$, one can write Yukawa couplings employing Higgs fields in the real $\underline{\mathbf{10}}_S$ and $\underline{\mathbf{120}}_S$ or the complex $\underline{\mathbf{126}}_S \oplus \overline{\mathbf{126}}_S$ representations. This leaves ample maneuvering space to accommodate the fermion masses, which a single Higgs representation would fail to explain due to phenomenologically unphysical Yukawa unifications. In particular, in order to generate a ”Majorana mass term” for the right-handed neutrinos and allow for a Type-I seesaw, the residual $U(1)$ symmetry needs to be broken. This can be achieved with a 16_S or a 126_S vev, with the difference that the second choice preserves a residual $\mathbb{Z}_2$ -symmetry, which can be understood as R -parity. The $\underline{\mathbf{5}}_S \oplus \overline{\mathbf{5}}_S$ Higgs fields of $SU(5)$ are embedded within a $\mathbf{10}_S$ representation of $SO(10)$. Finally, $SU(5)$ can be broken with the $\mathbf{24}_S$ contained within the $\mathbf{45}_S$ representation of $SO(10)$ (or a54or a210). Soft breaking terms a priori unify more completely at M_{GUT} than in $SU(5)$, since all MSSM fields are collected within $\mathbf{16}_F^f$ and $\mathbf{10}_S$. An even more ambitious unification pattern is possible in E_6 , with both $\mathbf{16}_F^f$ and $\mathbf{10}_S$ of $SO(10)$ collected within a 27 of E_6 , which we will not discuss here.

建议将所有夸克和轻子超场嵌入 $\mathbf{16}_F^f$ 的三代中，其中还包含相当于右手中微子的 $SU(5)$ 单态。我们可以发现 $\underline{16} \otimes \underline{16} = \underline{10} \oplus \underline{120} \oplus \underline{126}$，因此可以利用位于实 $\mathbf{10}_S$、$\mathbf{120}_S$ 表示或复 $\mathbf{126}_S \oplus \overline{\mathbf{126}}_S$ 表示中的希格斯场写出汤川耦合。这样就留有充足的调整空间来容纳费米子质量，若仅用单个希格斯表示则无法做到这一点，因为它会得到不符合唯象物理的汤川统一。特别是，为了给右手中微子生成“马约拉纳质量项”并实现 I 型跷跷板机制，需要破缺剩余的 $U(1)$ 对称性。这可以通过 16_S 或 126_S 真空期望值实现，区别在于第二种选择会保留剩余的 $\mathbb{Z}_2$ 对称性，该对称性可理解为 R 宇称。$SU(5)$ 的 $\underline{\mathbf{5}}_S \oplus \bar{\mathbf{5}}_S$ 希格斯场被嵌入 $SO(10)$ 的一个 $\mathbf{10}_S$ 表示中。最后，$SU(5)$ 可以通过包含在 $SO(10)$ 的 $\mathbf{45}_S$ 表示 (或 54 表示，或 210 表示) 中的 $\mathbf{24}_S$ 发生破缺。软破缺项先验地在 M_{GUT} 处比在 $SU(5)$ 中实现了更完全的统一，因为所有 MSSM 场都被收纳在 $\mathbf{16}_F^f$ 和 $\mathbf{10}_S$ 中。在 E_6 中可以实现更具野心的统一模式，$SO(10)$ 的 $\mathbf{16}_F^f$ 和 $\mathbf{10}_S$ 都被收纳进 E_6 的一个 27 表示中，我们在此不做讨论。

Matter stability is usually presented as the weak point of grand unified theories. Indeed, the interactions mediated by the $\left(3, 2, -\frac{5}{6}\right) \oplus \left(3, 2, \frac{5}{6}\right)$ gauge bosons contribute to effective dimension 6 baryon and lepton number-violating low-energy operators, which can mediate proton decay, via channels such as $p \to e^+\pi^0$. The typical associated lifetime scales like $\tau_p \sim M_{\mathrm{GUT}}^4/\left(\alpha_U^2 M_P^5\right)$, where M_P is the proton mass and $\alpha_U \equiv g_U^2/(4\pi)$ with g_U the gauge coupling at the unification scale. For a non-supersymmetric Grand Unification, the predicted lifetime tends to be incompatible with the corresponding experimental limit. In a supersymmetric context, the larger M_{GUT} apparently provides some breathing space. However, the existence of comparatively light scalar partners to the standard fermions alters the validity of the naive analysis, since it allows the emergence at M_{soft} of baryon and lepton number-violating operators of dimension 5 involving such fields, which in turn, below M_{soft} , alter the M_{GUT}^{-2} suppression of the contributions to dimension 6 operators mediating proton decay. Due to the structure of these operators, the proton decay modes that they funnel typically involve matter of higher-generation content, such as kaons, muons, muon and τ neutrinos. In practice, a loop suppression factor alleviates the contributions via such intermediate operators of dimension 5, so that these do not necessarily ruin the viability of supersymmetric Grand Unification: a detailed analysis is needed in each case. In fact, purely scalar dimension 4 operators at M_{soft} can also mediate proton decay, but contribute only at two-loop. In all this discussion, we have assumed the existence of a matter parity, which constrains baryon and lepton number-violating effects to appear in operators of higher dimensions (or involving scalar partners). The pattern of violation is modified if R -parity is not satisfied at M_{soft} , which we consider in the following section.

物质稳定性通常被认为是大统一理论的弱点。确实，由 $\left(3, 2, -\frac{5}{6}\right) \oplus \left(3, 2, \frac{5}{6}\right)$ 规范玻色子传递的相互作用会导致低能有效维数为 6 的重子数和轻子数破缺算符，这类算符可以通过 $p \to e^+\pi^0$ 之类的通道介导质子衰变。质子典型寿命的标度关系为 $\tau_p \sim M_{\mathrm{GUT}}^4/\left(\alpha_U^2 M_P^5\right)$，其中 M_P 是质子质量，而 $\alpha_U \equiv g_U^2/(4\pi)$，$g_U$ 是统一标度下的规范耦合。对于非超对称大统一，预言的质子寿命往往与实验给出的下限不符。在超对称框架下，更大的 M_{GUT} 表面上留出了缓冲空间。然而，标准费米子存在相对较轻的标量伙伴，这改变了 naive 分析的有效性，因为它允许在 M_{soft} 出现涉及这些场的 5 维重子数和轻子数破缺算符，而这类算符会在低于 M_{soft} 的能区改变对介导质子衰变的 6 维算符贡献的 M_{GUT}^{-2} 压低。由于这类算符的结构，它们产生的质子衰变模式通常包含更高代的物质，比如 K 介子、缪子、缪子和 τ 中微子。实际上，圈压低因子缓解了这类来自 5 维中间算符的贡献，因此它们并不一定会破坏超对称大统一的可行性: 具体情况都需要详细分析。事实上，M_{soft} 处的纯标量 4 维算符也可以介导质子衰变，但仅在两圈阶有贡献。在以上所有讨论中，我们都假设存在物质宇称，它将重子数和轻子数破缺效应限制在更高维算符 (或涉及标量伙伴的算符) 中出现。如果 R 宇称在 M_{soft} 不满足，破缺的模式就会发生改变，我们将在下一节讨论这种情况。

R-Parity-Violating Phenomenology

R 宇称破缺唯象学

So far, we have forbidden the terms in the second line of Eq. (33), and their soft breaking counterparts, by requesting the model to satisfy R -parity. The original motivation behind this choice was to forbid the emergence of baryon (in the last term of this equation: " UDD ") or lepton number-violating effects (in the three previous terms: " LH ," " LLE ," and " LQD "). As we noticed in the discussion concerning Grand Unification, however, R -parity does not forbid the emergence of non-renormalizable operators violating baryon and lepton number, as one would expect if the MSSM is only a low-energy effective field theory of some higher-energy dynamics; thus, it does not fully protect matter stability. We may therefore relax the assumption of R -parity conservation altogether, which results in significant alterations of the phenomenology. We refer the reader to Ref. [10].

迄今为止，我们通过要求模型满足 R 宇称，已经禁戒了式 (33) 第二行的项及其软破缺对应项。这一选择最初的动机是禁戒重子数 (对应式中最后一项:“UDD ”) 或轻子数破缺效应 (对应式中前三项:“LH ”、“LLE ”和“LQD ”) 产生。然而，正如我们在大统一相关讨论中指出的，如果 MSSM 仅仅是某高能动力学的低能有效场论，那么 R 宇称并不能禁戒破坏重子数和轻子数的不可重整算符出现；因此它无法完全保护物质稳定性。因此我们可以完全放宽 R 宇称守恒的假设，这会给唯象学带来显著改变，相关内容请读者参考文献 [10]。

Baryon number-violating processes, such as nucleon decay, obviously represent the first challenge to R -parity violation, as new physics at M_{soft} potentially contributes to such phenomena. Here, we stress that proton decay usually requires both lepton and baryon number violation, because leptons (electron, muon, neutrinos) are the only available fermions below the proton mass (in the absence of exotic particles), and that the angular momentum should be preserved in the transition. New discrete symmetries, such as baryon triality, lepton triality, or proton hexality, can be invoked to forbid nucleon decays. Pure baryon-violating processes such as di-nucleon decay or neutron-antineutron oscillations place further limits on couplings of UDD type, but they usually depend on the pattern of flavor violation as well, due to the antisymmetry of the baryon number-violating UDD operators with respect to down-quark generation indices. In the presence of a light exotic state, usually a bino, below the proton mass, proton decay can occur without lepton number violation. The baryon number-violating couplings thus appear as more immediately constrained and are often explicitly eliminated by the call to a symmetry.

重子数破缺过程 (例如核子衰变) 显然是对 R 宇称破缺的首要挑战，因为 M_{soft} 能标的新物理可能会对这类现象产生贡献。在此我们要强调，质子衰变通常同时需要轻子数和重子数破缺，因为 (不存在奇异粒子时) 轻子 (电子、缪子、中微子) 是质子质量以下仅有的可利用费米子，且跃迁过程需要角动量守恒。我们可以引入新的分立对称性 (例如重子三性、轻子三性或质子六性) 来禁戒核子衰变。纯重子破缺过程 (如双核子衰变或中子-反中子振荡) 对 UDD 类型的耦合给出更强的限制，但由于破坏重子数的 UDD 算符对于下夸克代指标是反对称的，这些限制通常还依赖味道破缺的模式。如果存在质量低于质子的轻奇异态 (通常是 bino)，质子衰变可以在不发生轻子数破缺的情况下发生。因此破坏重子数的耦合受到的约束更直接，通常会通过引入对称性被明确消除。

As we implied above, the bino state may be light if it is unstable, which is the case if R -parity is violated: cosmological and astrophysical bounds are indeed no longer applicable. If the bino is long-lived, its decays might be observed in far detectors, as currently deployed at the Large Hadron Collider. More generally, collider phenomenology may significantly change in the presence of R -parity-violating effects, as supersymmetric particles can now be produced as single resonances, while the missing energy signature is no longer guaranteed. Searches with single leptons, jets, and little missing energy may then prove viable alternative strategies. The lightest supersymmetric particle may still be a dark matter candidate if it is sufficiently long-lived to persist on cosmological timescales. In such a case, it needs to be neutral and satisfy conditions relative to its abundance at the time of recombination. Alternatively, a charged or colored particle with short lifetime is eligible as lightest supersymmetric particle for collider physics, without implications for cosmology.

> 正如我们上文暗示的，当 R 宇称破缺时，如果 bino 不稳定，它就可以很轻: 此时宇宙学和天体物理的限制确实不再适用。如果 bino 长寿命，它的衰变可以在大型强子对撞机现已部署的远端探测器中被观测到。更一般地说，存在 R 宇称破缺效应时，对撞机唯象学会发生显著改变: 超对称粒子现在可以作为单共振态产生，缺失横能量特征不再必然存在。因此，寻找单个轻子、喷注且缺失能量很小的过程可以成为可行的替代方案。如果最轻超对称粒子足够长寿，可以在宇宙学时间尺度上存在，它仍然可以作为暗物质候选者。在这种情况下，它需要是电中性的，并且满足复合时期丰度相关的条件。反之，对对撞机物理而言，寿命较短的带电或带色粒子也可以成为最轻超对称粒子，不会对宇宙学产生影响。

Given that R -parity is no longer present to distinguish between standard fields and their supersymmetric partners, the latter usually mix, such as leptons with uncolored gauginos and higgsinos or sleptons with Higgs bosons. This mixing should generally remain subleading so that standard particles retain their usual properties. It also entails some degree of ambiguity in the definition of the superfields: a convenient, though not mandatory, choice consists in requesting that the sneutrino fields do not take vevs. In addition, the emergence of neutrino masses is an interesting consequence of the mixing of neutrinos with neutral gauginos and higgsinos. At tree level, only one mass, scaling like $(\mu_i M_Z/\mu)^2/M_2$, develops; comparison with observed limits places bounds on $|\mu_i/\mu|, |\mu_i/M_2|$ of the order 10^{-5} . Loop corrections involving couplings of LLE or LQD type produce a second neutrino mass, opening the possibility for an explanation of neutrino oscillations in terms of pure R -parity-violating effects.

> 由于 R 宇称不再存在来区分标准场和它们的超对称伙伴，二者通常会发生混合，例如轻子与非色 gaugino 和 higgsino 混合，或者 slepton 与希格斯玻色子混合。这类混合通常必须是次要的，才能让标准粒子保留其通常的性质。混合也给超场的定义带来一定程度的歧义: 一种方便 (而非强制) 的选择是要求 sneutrino 场不获得真空期望值。此外，中微子与中性 gaugino 和 higgsino 的混合会产生中微子质量，这是一个有趣的结论。树图水平仅会产生一个标度为 $(\mu_i M_Z/\mu)^2/M_2$ 的质量；与观测限制对比得到对 $|\mu_i/\mu|, |\mu_i/M_2|$ 的约束约为 10^{-5} 。包含 LLE 或 LQD 类型耦合的圈修正会产生第二个中微子质量，使得可以纯粹通过 R 宇称破缺效应解释中微子振荡。

The a priori independent flavor- and CP-violating pattern of the trilinear R - parity-violating couplings can be constrained in low-energy observables. Such couplings indeed contribute at tree level (typically via a sfermion exchange) to precisely known observables, such as lepton and meson decays or particle-antiparticle oscillations, quark- or lepton-flavor transitions, the unitarity triangle, the anomalous magnetic dipole moments of leptons, the electric dipole moments of leptons and atoms, neutrinoless double beta decay, etc. A comprehensive coverage of these bounds is usually performed under simplifying assumptions, such as the dominance of a single R -parity-violating coupling or a pair of couplings, or in numerical form, as the diversity of available terms is otherwise difficult to efficiently tackle.

> 三线性 R 宇称破缺耦合的味和 CP 破缺模式本是独立的，可通过低能观测量对其进行约束。这类耦合确实会在树图阶 (通常通过超费米子交换) 对诸多精确已知的观测量产生贡献，例如轻子衰变、介子衰变、粒子-反粒子振荡、夸克或轻子味跃迁、幺正三角形、轻子反常磁偶极矩、轻子和原子电偶极矩、无中微子双贝塔衰变等等。通常会在简化假设下对这些界限进行全面梳理，例如假设单个 R 宇称破缺耦合或一对耦合占主导，或是以数值形式处理，否则可用项的多样性会导致难以高效处理。

From the perspective of Grand Unification, we have mentioned how the conservation/violation of R -parity could be related to $SO(10)$ breaking. In $SU(5)$, the R -parity-violating superpotential summarizes to

> 从大统一的角度来看，我们已经提到 R 宇称的守恒/破缺如何与 $SO(10)$ 破缺相关联。在 $SU(5)$ 中，R 宇称破缺的超势可归纳为

$$W_{SU(5)}^{\mathrm{RpV}} = \kappa_f \left(\overline{\overline{\mathbf{5}}}_F^f\right)^i \left(\overline{\mathbf{5}}_S\right)_i + \frac{1}{2}\lambda_{fgh}^{SU(5)} \left(\overline{\overline{\mathbf{5}}}_F^f\right)^i \left(\overline{\overline{\mathbf{5}}}_F^g\right)^j \left(\mathbf{10}^h\right)_{hj}. \tag{42}$$

In this case, trilinear couplings are unified, and the limits from proton decay make such couplings negligible at the electroweak scale. The coupling constants κ_i are also phenomenologically very small due to their implications for neutrino physics.

> 在这种情况下，三线性耦合会统一，质子衰变给出的限制使得这类耦合在电弱标度下可以忽略。由于对中微子物理的影响，耦合常数 κ_i 在唯象上也非常小。

As a summary of this phenomenological overview of supersymmetry/supergravity in particle physics, we have seen that very diverse types of effects could emerge within such theories, due to the rich field content and the a priori uncontrolled pattern of couplings inherited from their construction. Such models thus raise considerable challenge to model building for "natural" explanations of hierarchies, although these are now stabilized by supersymmetry (i.e., technically natural). The most simplistic model-building assumptions are usually incompatible with the experimental situation, leading to an increase in complexity in the field content and the parameter space, where one had originally wished a predictive and symmetry-constrained structure. Conversely, this diversity makes supersymmetry-inspired models a good laboratory to study and motivate non-standard effects in particle physics.

> 作为粒子物理学中超对称/超引力唯象概述的总结，我们已经看到，这类理论中会产生多种多样的效应，这源于其丰富的场内容，以及理论构建过程中继承来的、本无法控制的耦合模式。因此这类模型对“自然”解释等级问题的模型构建提出了相当大的挑战，尽管层级问题现在已经被超对称稳定 (即技术自然)。最简单的模型构建假设通常与实验情况不相容，导致场内容和参数空间的复杂度上升，而原本人们希望得到一个具有预言性、受对称性约束的结构。反过来说，这种多样性使得受超对称启发的模型成为研究和推动粒子物理中非标准效应的良好研究平台。

Cross-References

> ## 交叉引用

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有效量子引力的局域协变方法

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